Analysis of a Multi-Cell Converter under Unbalanced AC Source

Marcelo A. Perez*, Carlos R. Baier†, Jose R. Espinoza*, Jose R. Rodríguez†

*Departamento de Ingeniería Eléctrica, Universidad de Concepción
Casilla 53-C, Correo 3, Concepción, CHILE
Tel.: +56 41 203512, Fax.: +56 41 246999
(marceloperez,cbaiier,joseespi)@udec.cl

† Departamento de Ingeniería Electrónica, UTFSM
Casilla 110-V, Valparaíso, CHILE
Tel.: +56 32 654203, Fax.: +56 32 797469
jrp@elo.utfsm.cl

Abstract Multi-cell converters offer very useful characteristics for instance, it fulfills the input current harmonic content recommendations and presents a high power factor even at low levels of load at the ac system, and a reduced torque pulsations at the motor side. They also feature a high level of modularity and present a high reliability. Moreover, this converter does not produce electromagnetic interference or common-mode voltages and therefore, it is appropriate for large volt-ampere and high voltage motor drives. Surprising, the analysis under amplitude unbalances at the AC supply has not been reported in the literature. This work shows that the input multi-pulse transformer does not amplify the supply input unbalance. Thus, the secondary voltages feature identical unbalance, which in turn produces unbalanced secondary line currents. Similarly, the overall input currents feature an unbalance which is at most equal to the unbalance of the secondary line currents. It is also found that small voltages unbalances at the input of any cell produce large input line current unbalances. On the other hand, the current injected by the rectifier into the DC link produces a distorted DC voltage waveform, which in turn deteriorates the motor voltage. In this work, using symmetrical components, it is possible to analyze the effect in each of these stages by quantifying the amount of unbalance and distortion produced in the current and voltage waveforms. Besides, this analysis provides a design guideline to compute the DC link capacitor size necessary to reject off the effects of unbalances in the AC supply.

I. INTRODUCTION

Multi-cell converters are an effective alternative for the medium voltage drives, giving an improved quality of energy in both, motor and power supply sides. These converters also reduce torque pulsations and fully amplify the harmonic content recommendations of the input currents. In addition, they present a high power factor even at low levels of load [1]. The modularity characteristics, added to the transformer feeding connection to the network, gives a high reliability [2]. Moreover, these converters do not produce electromagnetic interference or common-mode voltage and are appropriate for large volt-ampere and high voltage motor drives [3]. The main causes and effects of the unbalanced ac systems in standard topologies, whereas the ways to quantify the unbalance using symmetric components have been studied [4]. One of the main consequences produced in rectifier bridges is the amplification of unbalanced currents [5], and the generation of second harmonic in the DC voltage [6]. When this rectifier is controlled, it is possible to use some mitigation techniques [7]. This is not the case in a multi-cell converter, because each cell has a diode bridge as an input stage. Because the system is usually designed to work with a balanced power supply [8], it is necessary to introduce some correction terms in the design to reduce the secondary effects [9]. This paper presents the analysis of the effects produced in the multi-cell converter that are fed with an unbalanced AC source. Specifically, the effects in the input current, DC voltage, output voltage, and load current are quantified. Finally, guidelines are given for designing the DC link capacitor considering unbalanced input voltages in order to reject off the secondary effects.

II. MODELING AND SIMULATION OF A MULTI-CELL CONVERTER UNDER UNBALANCED CONDITIONS

A. Multi-cell Converter Power Topology

The topology of a three level multi-cell converter is showed in Fig. 1(a). This structure uses N (N = 3 in this case) standard cells (as shown in Fig. 1(b)) connected in series to form one motor phase voltage. In order to maximize the output voltage, identical and synchronized fundamental frequency components are used. On the other hand, to reduce the presence of commutation harmonics in the load, the carrier signals are properly shifted [10]. The input current of each cell contains the 6k ± 1(k = 1, 2...) harmonics. Similarly to the motor side, the input currents of each cell are combined in the multi-pulse transformer and due to the phase angles of each group of the secondary transformer windings, these are added up generating a global input current with very low harmonic content. The power cell shown in Fig. 1(b) is composed by a three-phase diode-based rectifier, a capacitive DC link and a single-phase PWM inverter.

B. Model for Unbalanced Voltages

1) Unbalance quantification: In order to quantify the effects of the unbalance, the symmetric component decomposition is used. Considering the unbalanced input line voltages synchronized with voltage $v_{ab}$,

\[ v_{ab}(t) = V_{ab} \sin(\omega_{ab} t) \]

\[ v_{bc}(t) = V_{bc} \sin(\omega_{bc} t - 2\pi/3 + \phi_{bc}) \] (1)

\[ v_{ca}(t) = V_{ca} \sin(\omega_{ca} t + 2\pi/3 + \phi_{ca}) \]

because they add up zero, the angles in function of the amplitudes are,

\[ \phi_{bc} = - \arccos \left( \frac{V_{bc}^2 - V_{ab}^2 - V_{ca}^2}{2V_{bc}V_{ab}} \right) + \frac{2\pi}{3} \] (2)

\[ \phi_{ca} = \arccos \left( \frac{V_{ca}^2 - V_{bc}^2 - V_{ab}^2}{2V_{ca}V_{bc}} \right) - \frac{2\pi}{3} \] (3)
The amplitudes of positive and negative components are,

\[ V_p = \sqrt{V_{ab}^2 + V_{bc}^2 + V_{ca}^2 + 3K_v} \]  \\
\[ V_n = \sqrt{V_{ab}^2 + V_{bc}^2 + V_{ca}^2 - 3K_v} \]  \\

The unbalance factor is defined by the ratio of the negative to the positive sequence voltage, thus:

\[ U_u = \frac{V_n}{V_p} = \sqrt{\frac{V_{ab}^2 + V_{bc}^2 + V_{ca}^2 - \sqrt{3}K_v}{V_{ab}^2 + V_{bc}^2 + V_{ca}^2 + \sqrt{3}K_v}} \]  

Using the phase voltage, and \( v_a \) as reference for amplitude and angle, it is possible to draw the unbalance factor in terms of \( v_a \) and \( v_c \). Considering its variation only in magnitude. This unbalance factor is shown in Fig. 2(a) for given constant values. Every constant value is represented as a quadratic form in \( v_b \), \( v_c \) and mapped in line voltages as shown in Fig. 2(b). These line voltages can be composed like,

\[ V_{ab} = V \]  \\
\[ V_{bc} = a^2V - j\Delta C - \Delta \nu \]  \\
\[ V_{ca} = aV + j\Delta C + \Delta \nu \]  

where \( V \) is the amplitude component for a balanced and synchronized three-phase voltage, \( j\Delta C \) is an imaginary component which defines the difference between the balanced system and the center of circles. And \( \Delta \nu \) is the vector which amplitude is the radio of the circle and its angle defines the actual line voltage. Using the unbalance factor it is possible to calculate this components as,

in fasorial form,

\[ v_{ab} = V_{ab} \]  \\
\[ v_{bc} = \frac{V_{bc}^2 - V_{ab}^2 - V_{bc}^2}{2V_{ab}} - j\sqrt{K_v} \]  \\
\[ v_{bc} = \frac{V_{bc}^2 - V_{bc}^2 - V_{bc}^2}{2V_{ab}} + j\sqrt{K_v} \]  

with,

\[ K_v = 2(V_{ab}^2 + V_{bc}^2 + V_{ca}^2 + V_{ab}^2) - (V_{ab}^2 + V_{bc}^2 + V_{ca}^2) \]  

The symmetric component decomposition is expressed as,

\[ \begin{bmatrix} v_p \\ v_n \\ v_0 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} v_{ab} \\ v_{bc} \\ v_{ca} \end{bmatrix} \]  

Applying this transformation to the line voltages, the positive, negative and zero sequence voltages are given by,

\[ v_p = \frac{1}{2\sqrt{3}V_{ab}} (\sqrt{3}V_{ab} + \sqrt{K_v} + j(V_{ca} - V_{bc})) \]  \\
\[ v_n = \frac{1}{2\sqrt{3}V_{ab}} (\sqrt{3}V_{ab} - \sqrt{K_v} - j(V_{ca} - V_{bc})) \]  \\
\[ v_0 = 0 \]  

The quantification of the unbalance is done using the unbalance factor defined by the ratio of the negative to the positive sequence voltage. Thus, one can write,

\[ U_u = \frac{V_n}{V_p} = \sqrt{\frac{V_{ab}^2 + V_{bc}^2 + V_{ca}^2 - \sqrt{3}K_v}{V_{ab}^2 + V_{bc}^2 + V_{ca}^2 + \sqrt{3}K_v}} \]
\[ V = V_{ab} \]  
\[ \Delta C = \sqrt{3} U_c^2 \frac{1}{1 - U_b^2} V \]  
\[ \Delta V = \sqrt{3} U_b^2 \frac{1}{1 - U_c^2} V \]  

It is possible to note that all the amplitudes can be obtained with the unbalance factor amplitude, however the angle of the radius vector can be obtained using also the unbalance factor angle, which is usually not given.

2) Transformer analysis: Fig. 3 shows the input transformer configuration. In order to obtain the best harmonic cancellation in the input current it is necessary a phase angle equal to \( \phi = 60^\circ/N \). In the present analysis with \( N = 3 \), then the required phase angle is \( \phi = 20^\circ \). To obtain this phase angle in the secondary voltage it is necessary to add two primary voltages using the expressions,

\[ v^{s(-\phi)} = by_v + bx_A v_p \]  
\[ v^{s(+\phi)} = by_v + bx_A T v_p \]

where \( v_p \) is the primary voltage, \( v^{s(\phi)} \) is the secondary voltage shifted \( \phi \) degrees, \( b \) is the transformer ratio, and the matrix \( A \) is,

\[ A = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \]

this matrix allows the combination of two primary voltages to give the shifted secondary voltage. The desired secondary shift angle \( \phi \) is defined by the constants \( x \) and \( y \), and given by

\[ x = \frac{2}{\sqrt{3}} \sin(\phi), \quad y = \cos(\phi) - \sqrt{3}\sin(\phi), \]  

considering the primary line voltage in fasorial form,

\[ v_p = \begin{bmatrix} V_{ab} \\ \sqrt{v_{ab}^2 - v_{bc}^2 + v_{ca}^2} \\ \sqrt{v_{bc}^2 - v_{ab}^2 + v_{ca}^2} \end{bmatrix} \]  

then each secondary voltage is expressed as,

\[ v^{s(-20^\circ)} = b (y I + x A) v_p \]  
\[ v^{s(0^\circ)} = b v_p \]  
\[ v^{s(+20^\circ)} = b (y I + x A^T) v_p \]

for symmetry reasons, we consider one secondary in \(-20^\circ\), the second one in phase with the primary, and the last one in \(+20^\circ\).

Using as primary voltages the line voltages expressed in fasorial form, the secondary voltages amplitudes are

\[ v^{s(-20^\circ)} = b \frac{1}{2V_{ab}} \sqrt{k_1 V_{ab}^2 + k_2 V_{bc}^2 + k_3 V_{ca}^2} \]  
\[ v^{s(0^\circ)} = b \frac{1}{2V_{ab}} \sqrt{k_1 V_{bc}^2 + k_2 V_{ab}^2 + k_3 V_{ca}^2} \]  
\[ v^{s(+20^\circ)} = b \frac{1}{2V_{ab}} \sqrt{k_1 V_{ca}^2 + k_2 V_{bc}^2 + k_3 V_{ab}^2} \]  

with,

\[ k_1 = (x + y)(2x + y), \quad k_2 = -x(x + y), \quad k_3 = x(2x + y) \]

Thus the amplitude of the positive and negative sequences of the secondary voltages are,

\[ V_p^{s(-20^\circ)} = b \sqrt{3x^2 + 3xy + y^2} V_p \]  
\[ V_n^{s(-20^\circ)} = b \sqrt{3x^2 + 3xy + y^2} V_n \]  
\[ V_p^{s(0^\circ)} = b V_p \]  
\[ V_n^{s(0^\circ)} = b V_n \]  
\[ V_p^{s(+20^\circ)} = b \sqrt{3x^2 + 3xy + y^2} V_p \]  
\[ V_n^{s(+20^\circ)} = b \sqrt{3x^2 + 3xy + y^2} V_n \]

the term \( 3x^2 + 3xy + y^2 \) is always unity, so every symmetrical component in the secondary is equal to the correspondent symmetrical component in the primary scaled by the transformation ratio. Therefore the unbalance factor for each secondary voltage is the same and equal to the unbalance factor in the primary.

Considering the secondary currents,

\[ i^{s(-20^\circ)}, i^{s(0^\circ)}, i^{s(+20^\circ)} \]

The primary currents contributed by each secondary are,

\[ i^{P\alpha} = b (y I + x A) i^{s(\alpha)} \]

where \( i^{P\alpha} \) is the current from the secondary shifted an angle \( \alpha \). In this case, as the voltages, each current in the primary features the same unbalance factor than the corresponding secondary.

The total primary current is given by,

\[ i^P = i^{P\alpha} + i^{P\beta} + i^{P\gamma} \]
is the conducting time, \( I \) is the number of pulses, \( I \) is the current
\[ V = \max \left( V_c \right) - \min \left( V_c \right) \]
\[ n_p \cdot \frac{I_{max}}{2} \cdot T_c = I_o T \]
and $T$ is the period. The conducting time can be calculated for each conducting sector as,

$$T_c = K \frac{T}{6}$$

(48)

The current amplitude is given by,

$$I_{\text{max}} = \frac{12I_o}{n_p K}$$

(49)

For 6 pulses mode, $K \approx \frac{2}{3}$, then

$$I_{\text{max6}} \approx 3I_o$$

(50)

For 4 pulses mode, $K \approx \frac{3}{4}$, then

$$I_{\text{max4}} \approx 4I_o$$

(51)

For 2 pulses mode, $K \approx 1$, then

$$I_{\text{max2}} \approx 6I_o$$

(52)

Considering $V_{\Delta} \geq \Delta V(U) \geq V_{r6}$ the fundamental current amplitude, referenced to $i_a$, is given by,

$$I_a = \sqrt{T_c} I_{\text{max}}, \quad I_b = \sqrt{T_c/3} I_{\text{max}}, \quad I_c = \sqrt{T_c/3} I_{\text{max}}$$

(53)

The unbalance factor for this current is

$$U_i = 50\%$$

(54)

Considering $V_{r2} \geq \Delta V(U) \geq V_{r4}$ the fundamental current amplitude, referenced to $i_a$, is given by,

$$I_a = \sqrt{T_c} I_{\text{max}}, \quad I_b = \sqrt{T_c} I_{\text{max}}, \quad I_c = 0$$

(55)

The unbalance factor for this current is

$$U_i = 100\%$$

(56)

in Fig. 8 is shown the relationship between the unbalance factor in the input line voltage and the unbalance factor in the input current.

Defining

$$M_{n_p} = \frac{I_o}{\sqrt{3} V_{n_p} f C_{dc}}$$

(57)

and

$$U_{v,n_p} = \frac{1}{2M_{n_p}} \left(1 - \sqrt{1 - 4M_{n_p}}\right)$$

(58)

in Fig. 9 is shown the levels of unbalance factor in the input line voltage in terms of DC capacitance.

4) DC-link Analysis: The injected current from the rectifier to the DC link contains even harmonics. Using the Fourier coefficients it is possible to calculate the amplitude of these harmonics.

In the 6 pulses mode there is a $6^{th}$ harmonic with magnitude,

$$I_6 = \frac{3}{2} I_o$$

(59)

in this mode, there are negligible $2^{nd}$ and $4^{th}$ harmonics. The amplitude of the voltage reflected by this current on the DC voltage is,

$$V_{dc6} = \frac{I_o}{8\pi f C_{dc}}$$

(60)

In the 4 pulses mode appears a $2^{nd}$, $4^{th}$ and $6^{th}$ harmonics with amplitudes

$$I_2 = I_4 = \frac{3}{4} I_o, \quad I_6 = \frac{3}{2} I_o$$

(61)

The voltages reflected on the DC link are

$$V_{dc2} = \frac{I_o}{4\pi f C_{dc}}, \quad V_{dc4} = \frac{3I_o}{32\pi f C_{dc}}, \quad V_{dc6} = \frac{I_o}{8\pi f C_{dc}}$$

(62)
The maximum dc voltage variation assuming all the three voltages in phase is,

$$\Delta V_{dc} = \frac{15I_o}{32\pi fC_{dc}}$$  \hspace{1cm} (63)

In the 2 pulses mode appears a 2\textsuperscript{nd}, 4\textsuperscript{th} and 6\textsuperscript{th} harmonics with amplitudes

$$I_2 = 2I_o \quad I_4 = \frac{3}{2}I_o \quad I_6 = I_o$$  \hspace{1cm} (64)

and the voltages reflected on the DC link are,

$$V_{dc2} = \frac{I_o}{2\pi fC_{dc}} \quad V_{dc4} = \frac{3I_o}{16\pi fC_{dc}} \quad V_{dc6} = \frac{I_o}{12\pi fC_{dc}}$$  \hspace{1cm} (65)

Thus, the maximum dc voltage variation in this case is,

$$\Delta V_{dc} = \frac{37I_o}{48\pi fC_{dc}}$$  \hspace{1cm} (66)

All this cases are shown in Fig. 7. On the other hand, from the load, it is injected a current at 2\textit{f}_o, with amplitude

$$I_{L,2} = \frac{I_o}{\cos(\phi)}$$  \hspace{1cm} (67)

Assuming the output frequency equal to the input, the voltage imposed by this current is,

$$V_{dc2} = \frac{I_o}{\cos(\phi)4\pi fC_{dc}}$$  \hspace{1cm} (68)

The worst operational condition is when the rectifier works in 2 pulses mode, the voltage variation is the sumatories of (66) and (68), thus

$$\Delta V_{dc} = \frac{I_o}{4\pi fC_{dc}} \left(3 + \frac{1}{\cos(\phi)}\right)$$  \hspace{1cm} (69)

In order to minimize the DC voltage variation, the DC link capacitor design should consider these latest harmonics. Otherwise, it can distort the DC voltage and consequently the output voltage.

5) Load Analysis: Considering the DC voltage,

$$v_{dc} = V_{dc0} + V_{dc2} \sin(2\omega t) + V_{dc4} \sin(4\omega t) + V_{dc6} \sin(6\omega t)$$  \hspace{1cm} (70)

the output voltage contains the modulation of the DC voltage with the inverter modulating signal at the output frequency.

$$v_o = m_o \left[V_{dc0} + V_{dc2} \sin(\omega t) + V_{dc4} \sin(3\omega t) + V_{dc6} \sin(5\omega t)\right]$$  \hspace{1cm} (71)

The load current contains the same harmonics, attenuated by the load impedance.

C. Simulation

The system is simulated in the balanced condition in Fig. 10, and in the unbalanced condition in Fig. 11, the simulation parameters are shown in the Table I.

| TABLE I: SIMULATION PARAMETERS |
|-----------|-------------|-------------|-------------|
| Condition | Balanced    | Unbalanced  | Improved DC design |
| \(U_v(\%)\) | 0           | 3.125       | 3.125       |
| \(C\) | 3300\mu F | 3300\mu F | 6100\mu F |
| \(V_{dc}\) | 1\%V       | 1.6%        | 1%          |
| \(U_i(\%)\) | 14         | 100         | 100         |

Fig. 10. Simulation in balanced condition

Fig. 11. Simulation in unbalanced condition
The input filter parameters are $L_s = 1\text{mH}$ and $R_s = 0.1\Omega$, the input and output frequency are 50Hz, and the load parameters are $P_o = 2\text{kw}$ and $p.f. = 0.86$.

### III. PASSIVE COMPONENTS DESIGN GUIDELINES

The DC capacitor design for a balanced system considers the second harmonic current injected by the inverter [11],

$$I_{\text{DClink}} = \frac{V_o I_o}{2V_{\text{dc}}} - \frac{V_o I_o}{2V_{\text{dc}} \cos(\phi)} \cos(2\omega_o + \phi),$$

and for a given DC voltage variation the value of the DC link capacitor is,

$$C_{\text{dc}} = \frac{I_o}{4\pi f V_{\text{dc}} \cos(\phi)}$$

In the case of unbalance, the current in the DC link is the summation of the current injected by the inverter and the current injected by the rectifier. Thus the voltage variation is,

$$V_{\text{dc}} = \frac{I_o}{4\pi f V_{\text{dc}} \cos(\phi)} \left(3 + \frac{1}{\cos(\phi)}\right)$$

and thus the capacitance is,

$$C_{\text{dc}} = \frac{I_o}{4\pi f V_{\text{dc}} \cos(\phi)} (3\cos(\phi) + 1)$$

The simulations in Fig. 12 show the unbalanced system with the DC capacitor design for unbalanced conditions. The DC link voltage is clearly improved and consequently the output voltage Fig. 12(b). The output current does not show a major improvement due to the inductive load Fig. 12(c). The rectifier input current also does not present a major improvement Fig. 12(a).

### IV. CONCLUSION

Unbalanced supply voltages affect the different stages of multi-cell converter in many ways. This paper identifies and evaluates these several effects and proposes a mitigating alternative. In fact, each secondary voltage of the multi-pulse transformer results unbalanced in magnitude and phase; however, the unbalance factor is identical to the primary voltage. As a result, the rectifier produces an unbalanced input current including odd harmonics. The phase unbalance present in the current is not negligible and can produce distorted and unbalanced overall input currents. The current injected into the DC link from the rectifier contains even harmonics components. These harmonic components generate even voltage harmonics in the DC link voltage and can reect this distortion to the output voltage across the single phase inverter. Generally, the load is very inductive and its current is not affected by the distortion at the output voltage. One form of mitigating the distortion effects in the DC link voltage is redesigning the DC capacitor. To do so, this paper also presents a design guideline for the DC capacitor that includes the unbalance factor of the input currents.

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