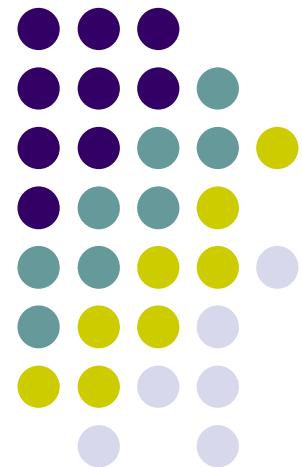
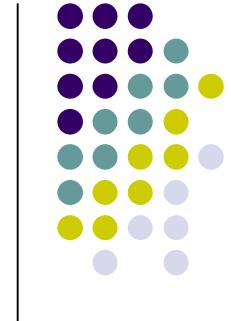


# Procesamiento Digital de Imágenes

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Pablo Roncagliolo B.  
Nº 08



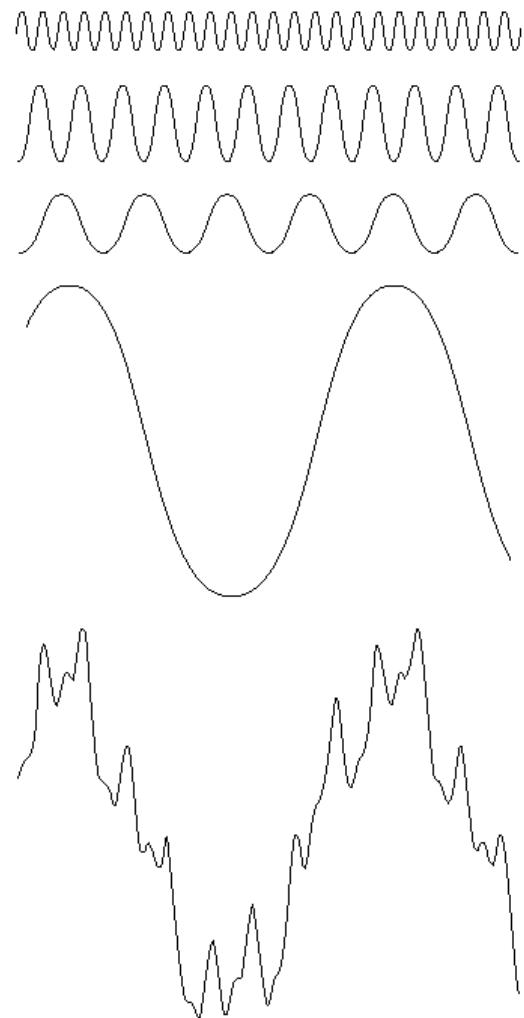


# **TRATAMIENTO DE IMÁGENES**

## **EN EL DOMINIO DE LAS FRECUENCIAS**

# Tratamiento de Imágenes:

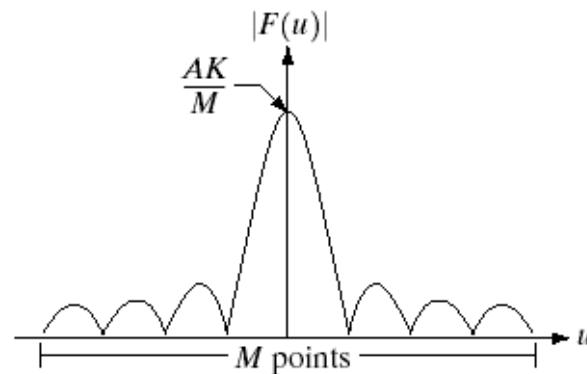
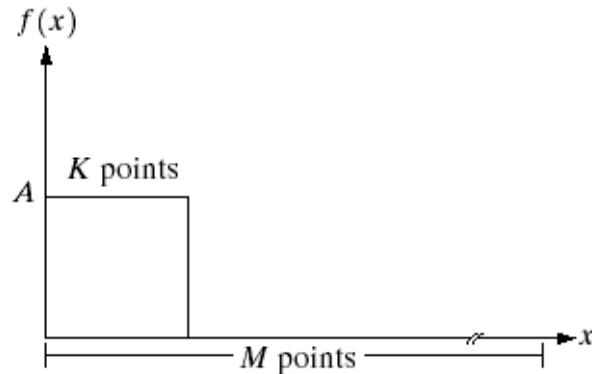
## Dominio Frecuencias:



**FIGURE 4.1** The function at the bottom is the sum of the four functions above it. Fourier's idea in 1807 that periodic functions could be represented as a weighted sum of sines and cosines was met with skepticism.

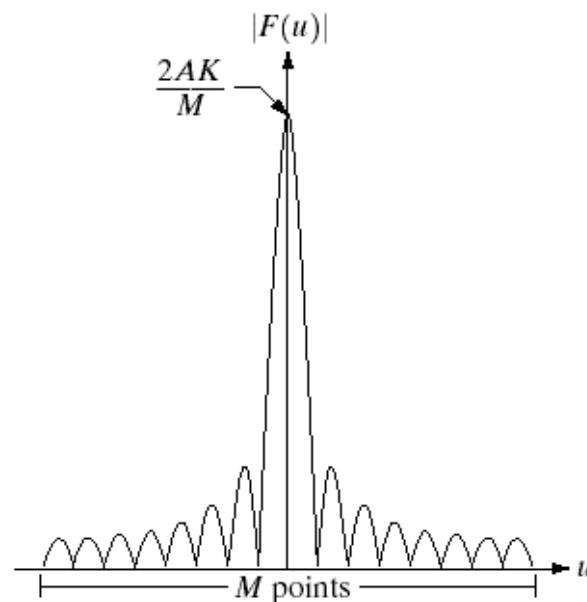
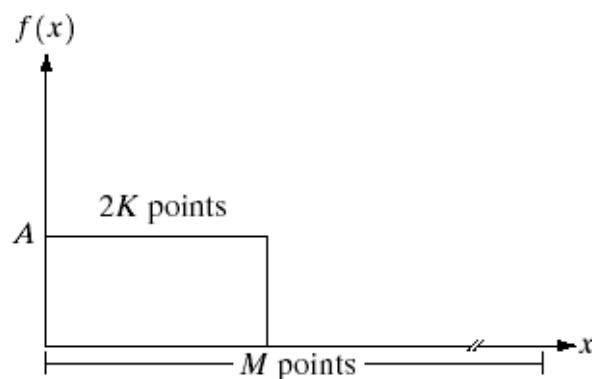
# Tratamiento de Imágenes:

## Dominio Frecuencias: “ejemplo transformada Fourier 1D”



a	b
c	d

**FIGURE 4.2** (a) A discrete function of  $M$  points, and (b) its Fourier spectrum. (c) A discrete function with twice the number of nonzero points, and (d) its Fourier spectrum.



# Tratamiento de Imágenes: Dominio Frecuencias: “ejemplo transformada Fourier 2D”

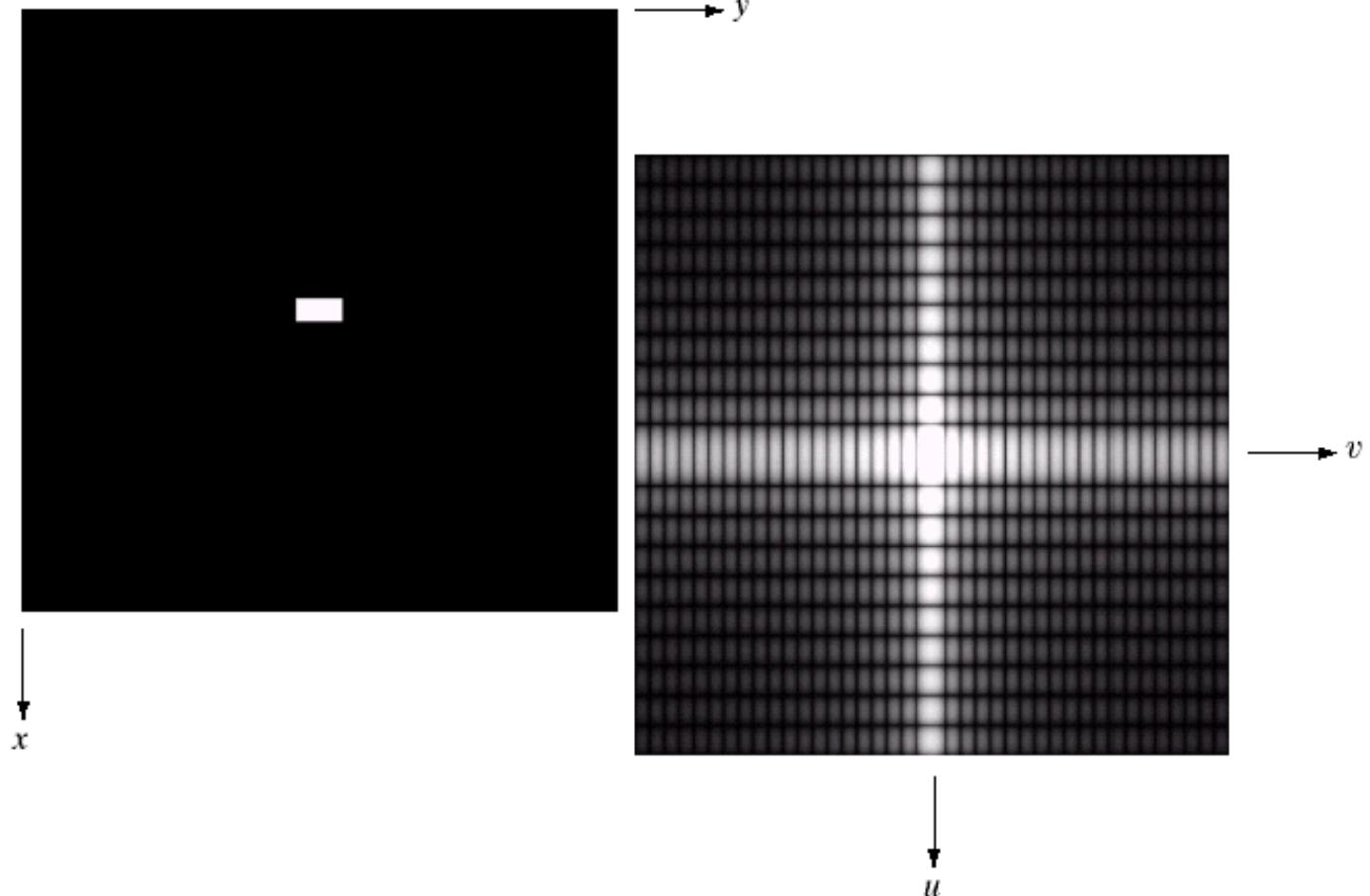


a b

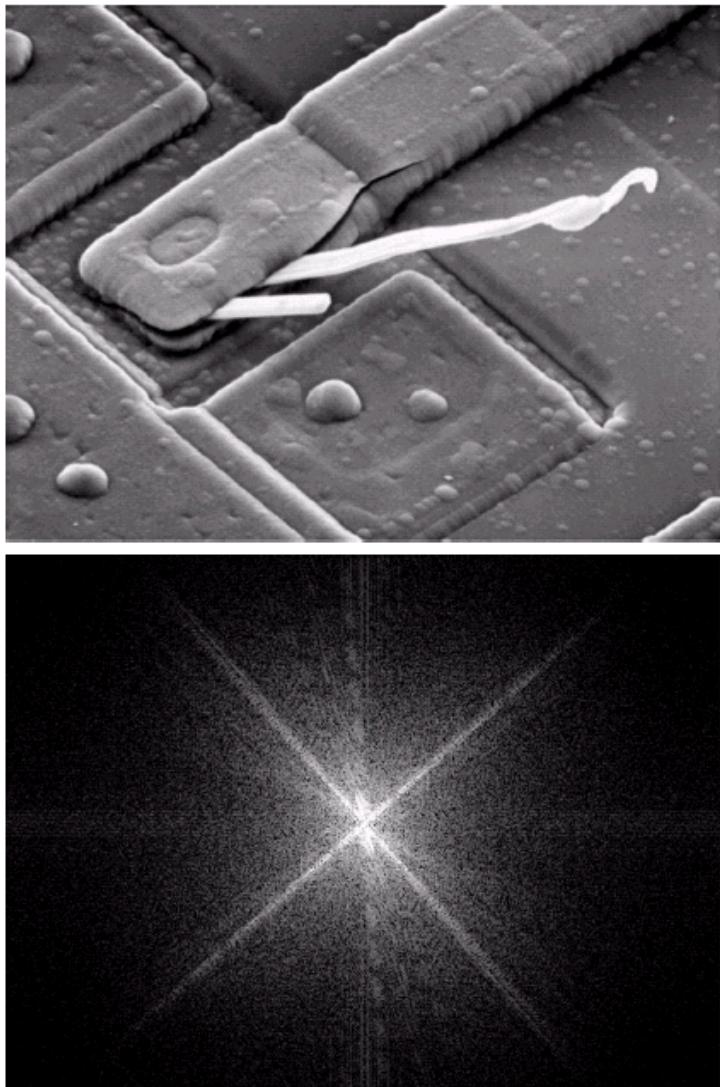
**FIGURE 4.3**

(a) Image of a  $20 \times 40$  white rectangle on a black background of size  $512 \times 512$  pixels.

(b) Centered Fourier spectrum shown after application of the log transformation given in Eq. (3.2-2). Compare with Fig. 4.2.



# Tratamiento de Imágenes: Dominio Frecuencias: “ejemplo transformada Fourier”

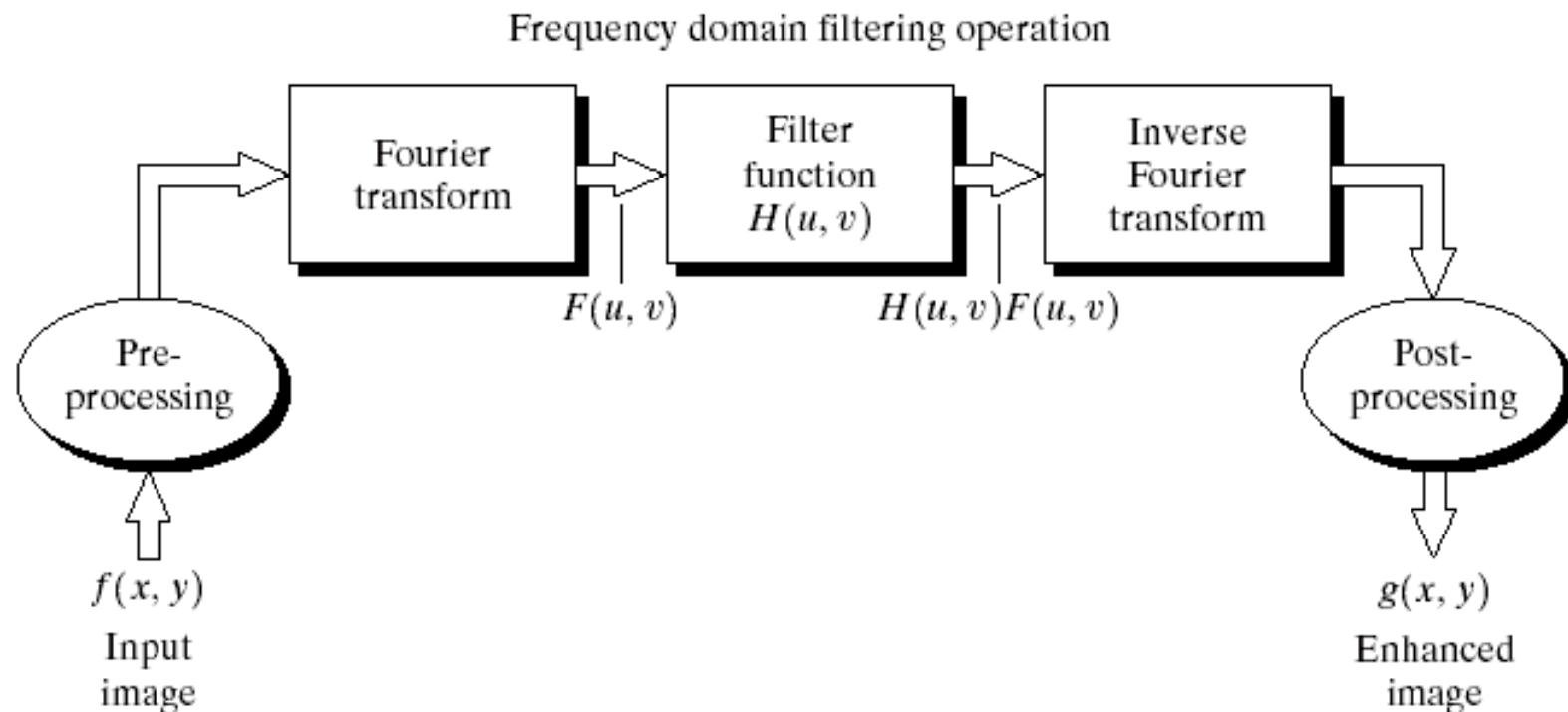


a  
b

**FIGURE 4.4**  
(a) SEM image of a damaged integrated circuit.  
(b) Fourier spectrum of (a).  
(Original image courtesy of Dr. J. M. Hudak, Brockhouse Institute for Materials Research, McMaster University, Hamilton, Ontario, Canada.)

# Tratamiento de Imágenes:

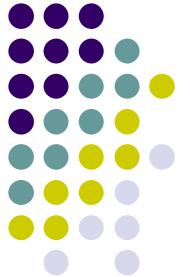
## Dominio Frecuencias: “proceso de filtrado”



**FIGURE 4.5** Basic steps for filtering in the frequency domain.

# Tratamiento de Imágenes:

## Dominio Frecuencias: “eliminación de continua”

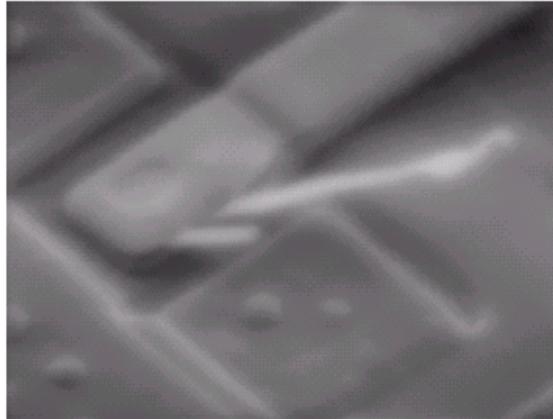
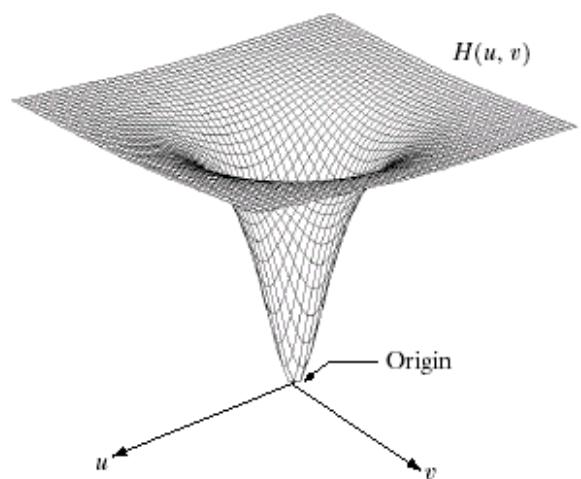
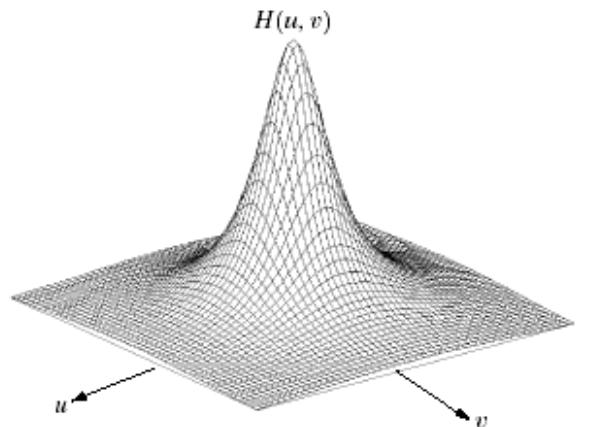
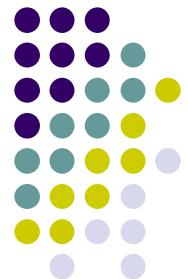


**FIGURE 4.6**

Result of filtering the image in Fig. 4.4(a) with a notch filter that set to 0 the  $F(0, 0)$  term in the Fourier transform.



# Tratamiento de Imágenes: Dominio Frecuencias: “Funciones de Filtros (LP y HP)”



a	b
c	d

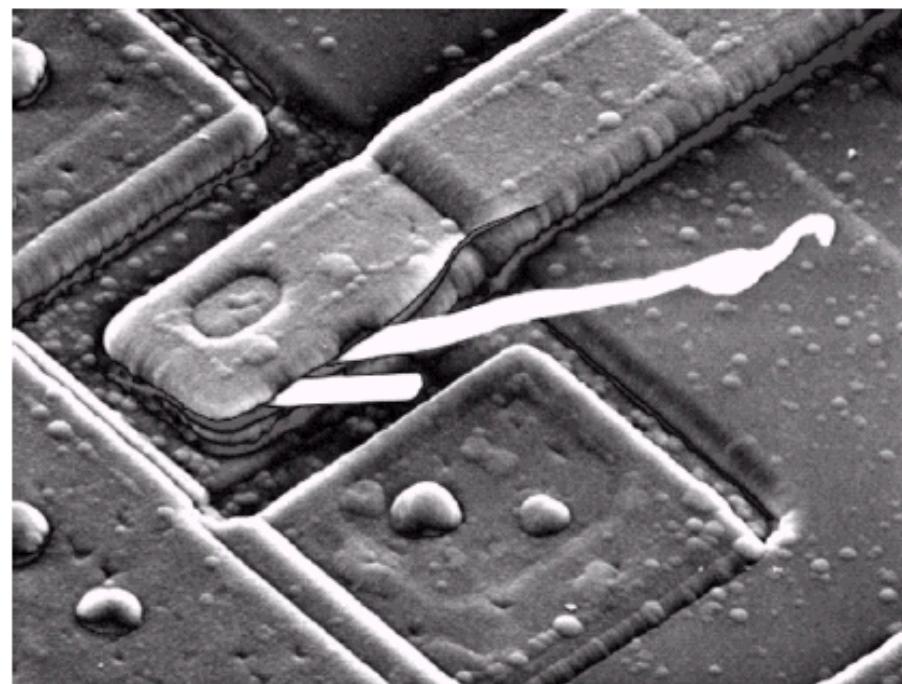
**FIGURE 4.7** (a) A two-dimensional lowpass filter function. (b) Result of lowpass filtering the image in Fig. 4.4(a).  
(c) A two-dimensional highpass filter function. (d) Result of highpass filtering the image in Fig. 4.4(a).

# Tratamiento de Imágenes: Dominio Frecuencias:



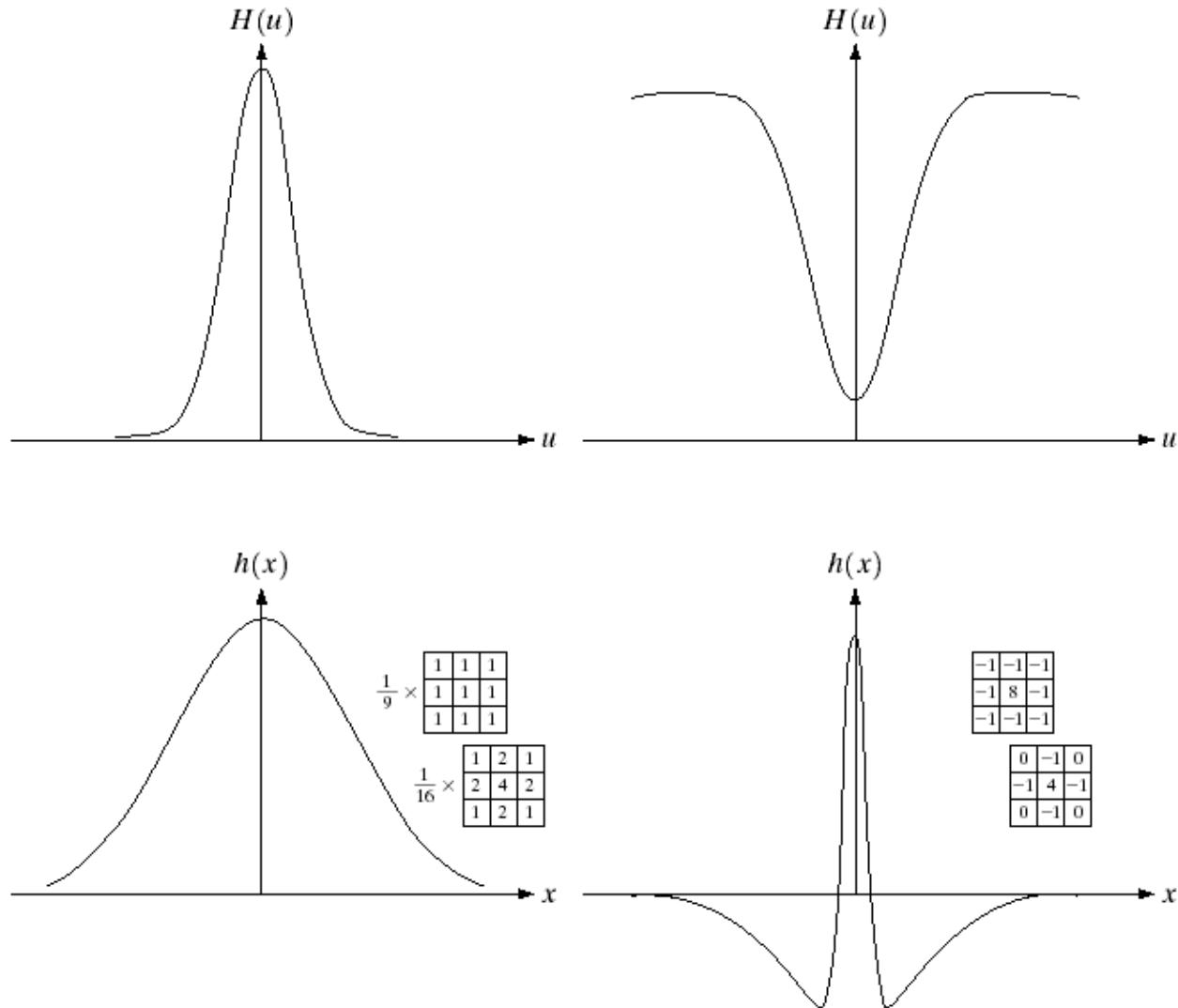
**FIGURE 4.8**

Result of highpass filtering the image in Fig. 4.4(a) with the filter in Fig. 4.7(c), modified by adding a constant of one-half the filter height to the filter function. Compare with Fig. 4.4(a).



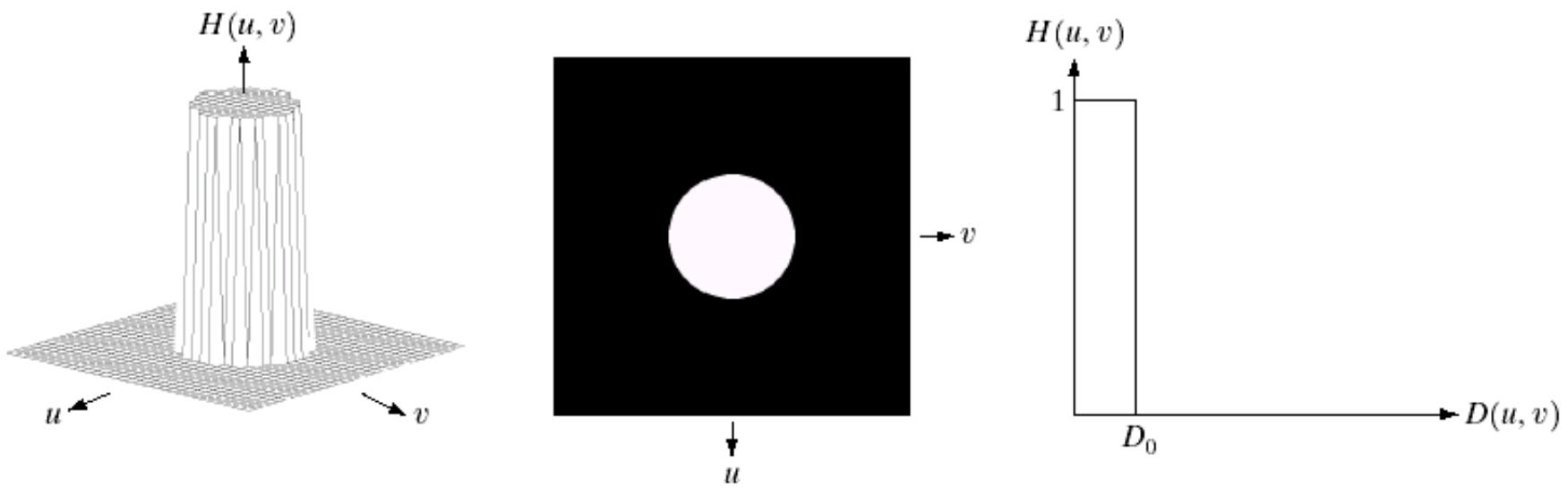
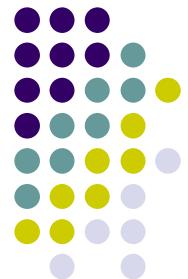
# Tratamiento de Imágenes:

**Dominio Frecuencias:** “relación entre filtro espaciales y filtros en frecuencia”



**FIGURE 4.9**  
 (a) Gaussian frequency domain lowpass filter.  
 (b) Gaussian frequency domain highpass filter.  
 (c) Corresponding lowpass spatial filter.  
 (d) Corresponding highpass spatial filter. The masks shown are used in Chapter 3 for lowpass and highpass filtering.

# Tratamiento de Imágenes: Dominio Frecuencias: “Filtro ideal en frecuencia”

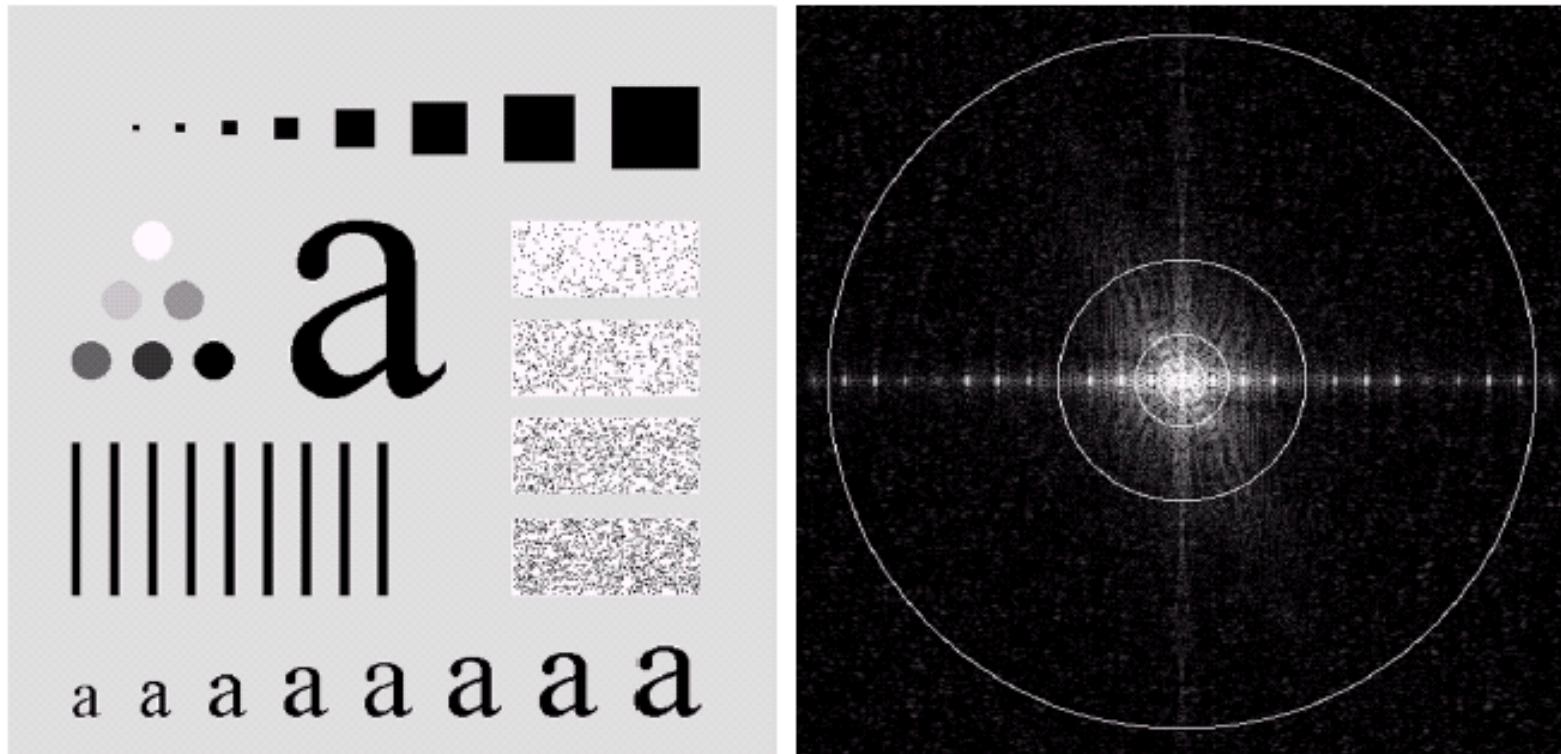
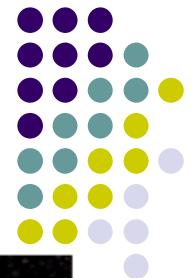


a b | c

**FIGURE 4.10** (a) Perspective plot of an ideal lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.

# Tratamiento de Imágenes:

**Dominio Frecuencias:** ¿dónde está la imagen en la FFT? ¿dónde está la información?

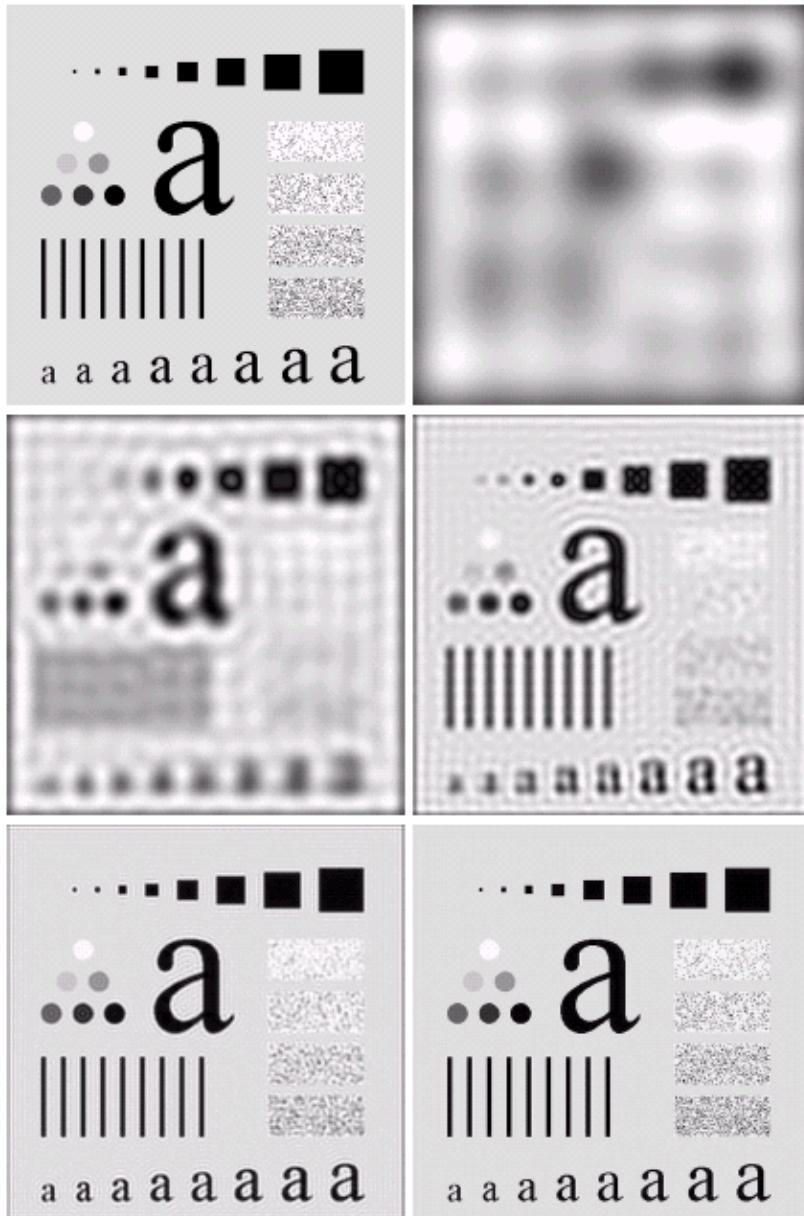
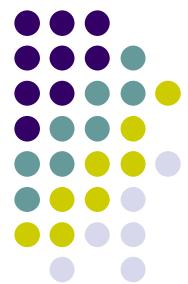


a | b

**FIGURE 4.11** (a) An image of size  $500 \times 500$  pixels and (b) its Fourier spectrum. The superimposed circles have radii values of 5, 15, 30, 80, and 230, which enclose 92.0, 94.6, 96.4, 98.0, and 99.5% of the image power, respectively.

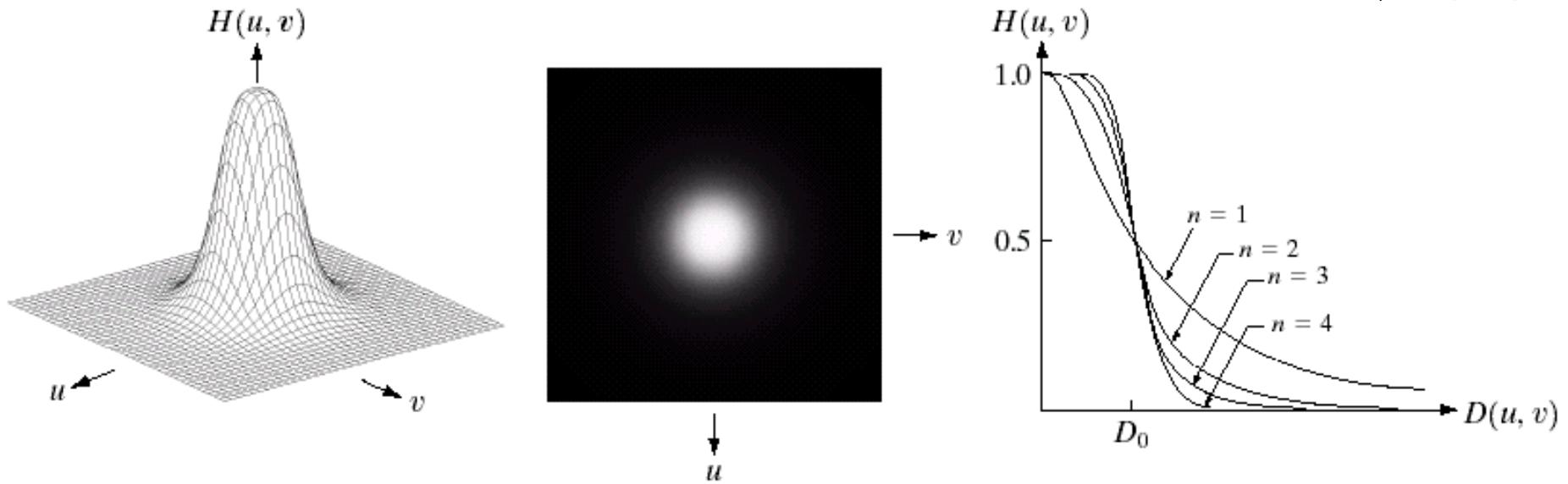
# Tratamiento de Imágenes:

## Dominio Frecuencias: “efecto de un filtro ideal”



a b      **FIGURE 4.12** (a) Original image. (b)–(f) Results of ideal lowpass filtering with cutoff frequencies set at radii values of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). The power removed by these filters was 8, 5.4, 3.6, 2, and 0.5% of the total, respectively.

# Tratamiento de Imágenes: Dominio Frecuencias: “Filtros no ideales”



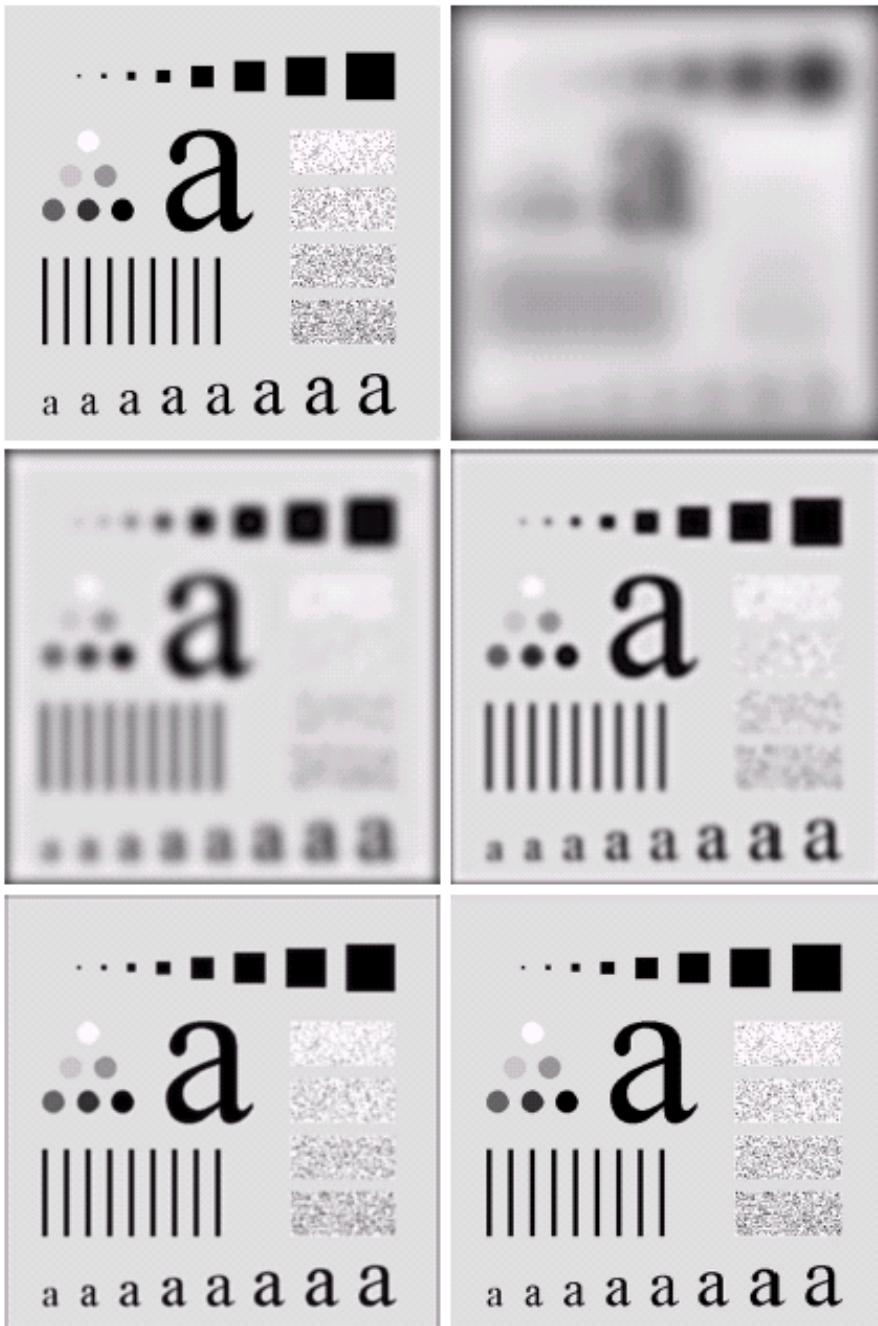
a b c

**FIGURE 4.14** (a) Perspective plot of a Butterworth lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.



## Tratamiento de Imágenes:

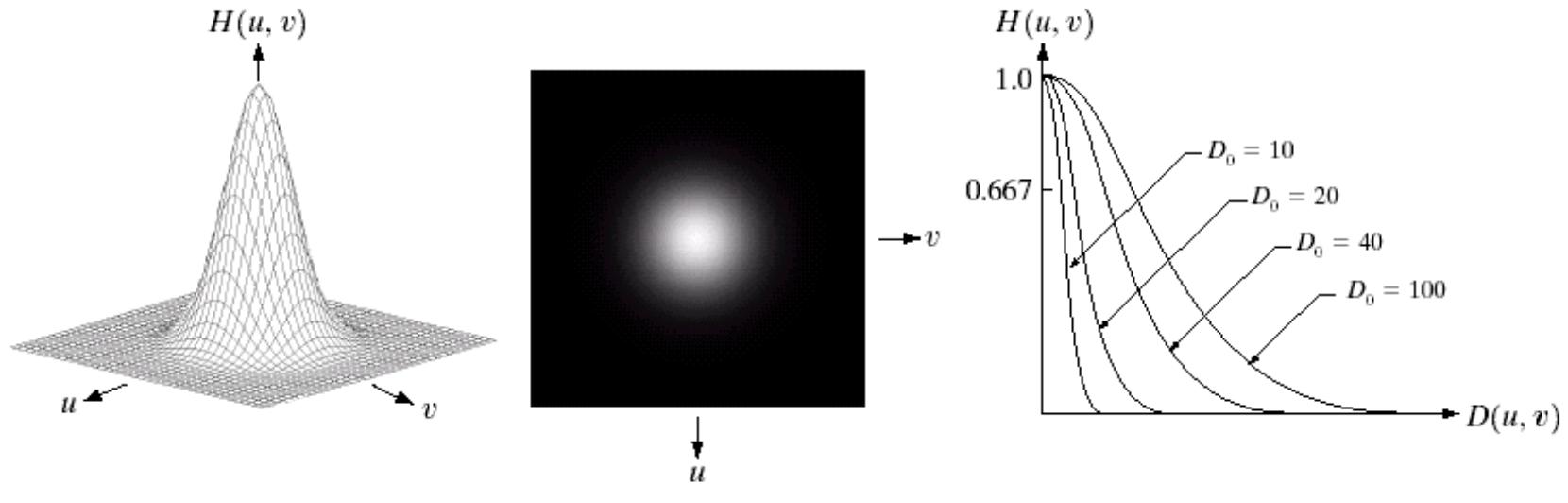
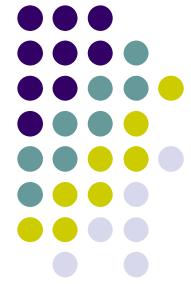
Dominio Frecuencias:  
“efecto de filtros Butterworth”



a b  
c d  
e f

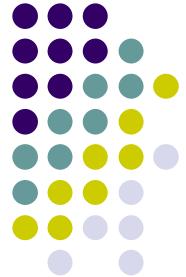
FIGURE 4.15 (a) Original image. (b)–(f) Results of filtering with BLPFs of order 2, with cutoff frequencies at radii of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). Compare with Fig. 4.12.

# Tratamiento de Imágenes: Dominio Frecuencias: “Filtro Gaussiano”



a b c

**FIGURE 4.17** (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of  $D_0$ .



## Tratamiento de Imágenes: Dominio Frecuencias: “Efectos de filtros Gaussianos”

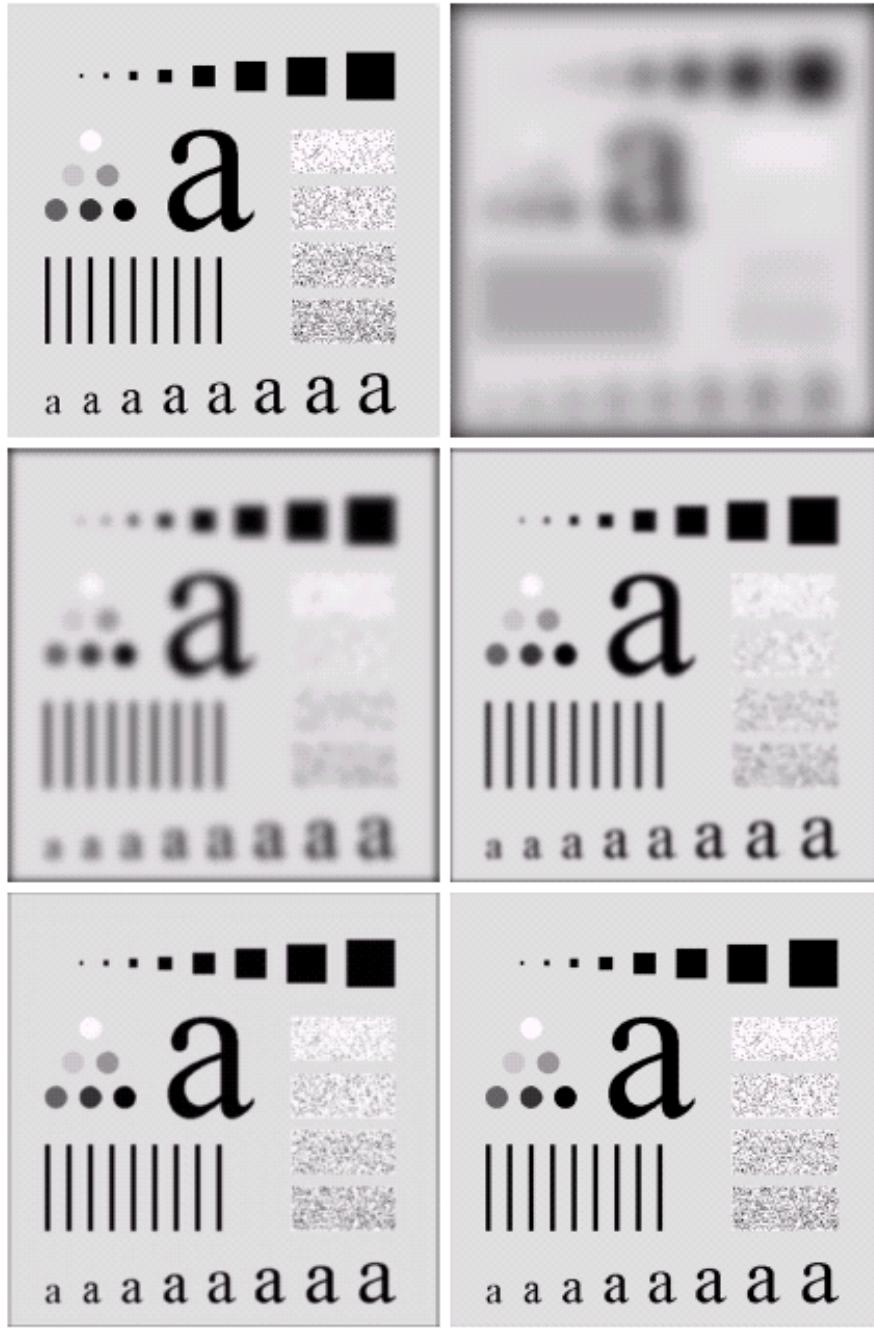
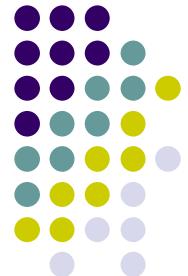


FIGURE 4.18 (a) Original image. (b)–(f) Results of filtering with Gaussian lowpass filters with cutoff frequencies set at radii values of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). Compare with Figs. 4.12 and 4.15.

a b  
c d  
e f

# Tratamiento de Imágenes:

## Dominio Frecuencias: Ej. Aplicación de Filtros

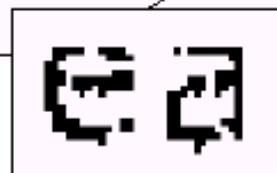


a b

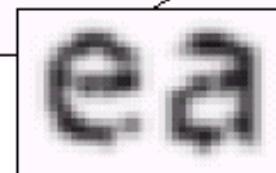
**FIGURE 4.19**

(a) Sample text of poor resolution (note broken characters in magnified view).  
(b) Result of filtering with a GLPF (broken character segments were joined).

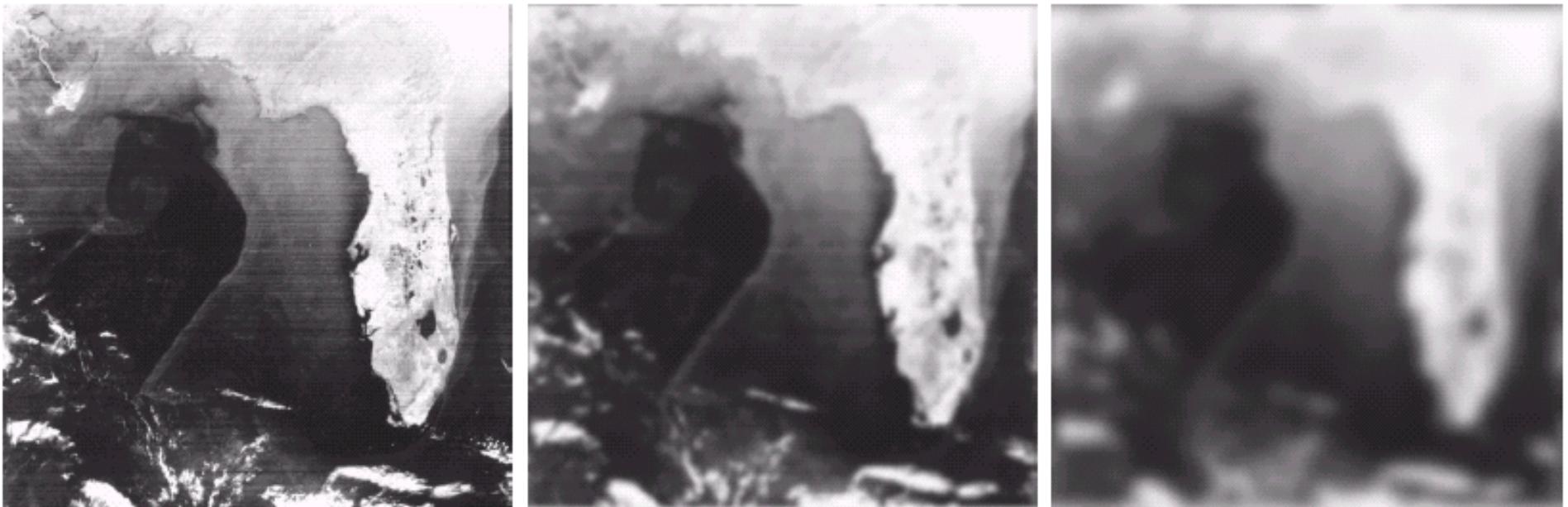
Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.



Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.



# Tratamiento de Imágenes: Dominio Frecuencias:

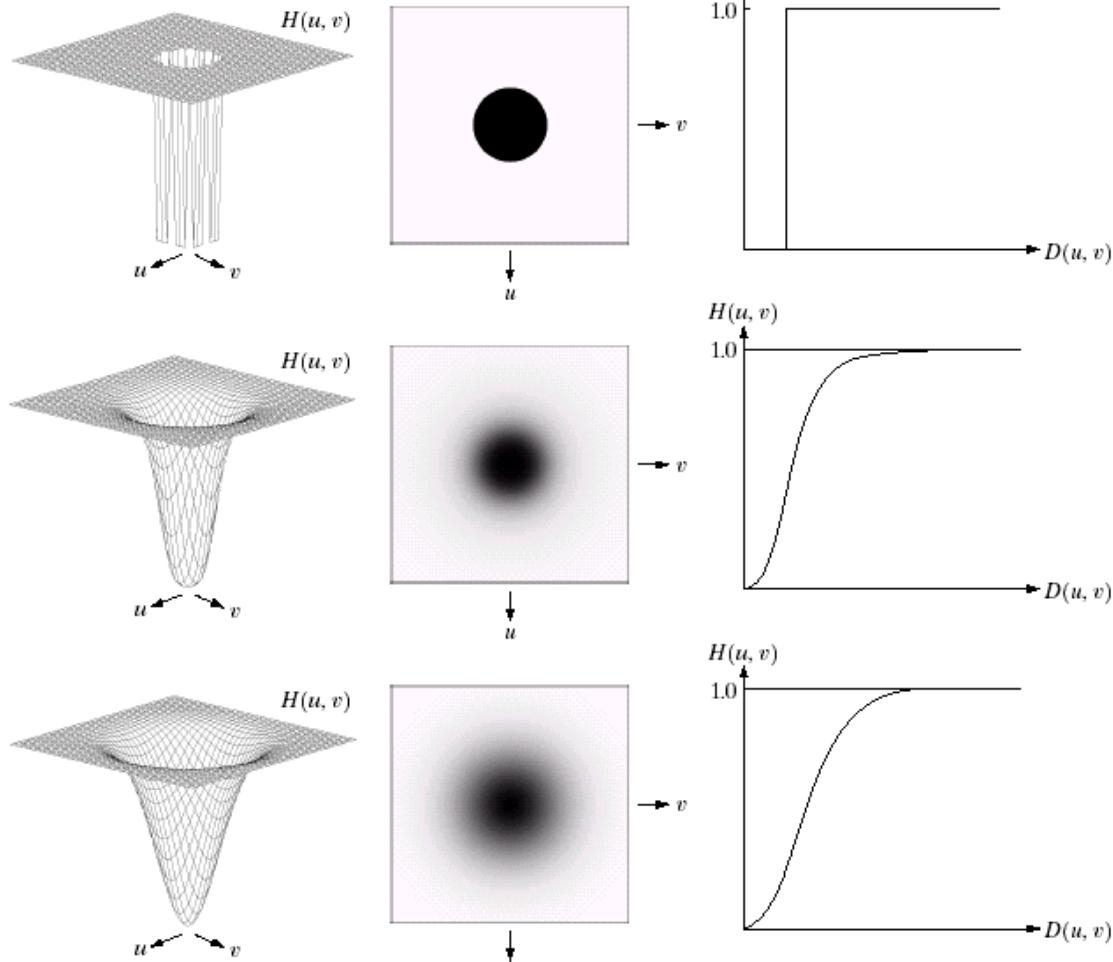


a b c

**FIGURE 4.21** (a) Image showing prominent scan lines. (b) Result of using a GLPF with  $D_0 = 30$ . (c) Result of using a GLPF with  $D_0 = 10$ . (Original image courtesy of NOAA.)

# Tratamiento de Imágenes:

## Dominio Frecuencias: “Filtros pasa altos en frecuencia”



a	b	c
d	e	f
g	h	i

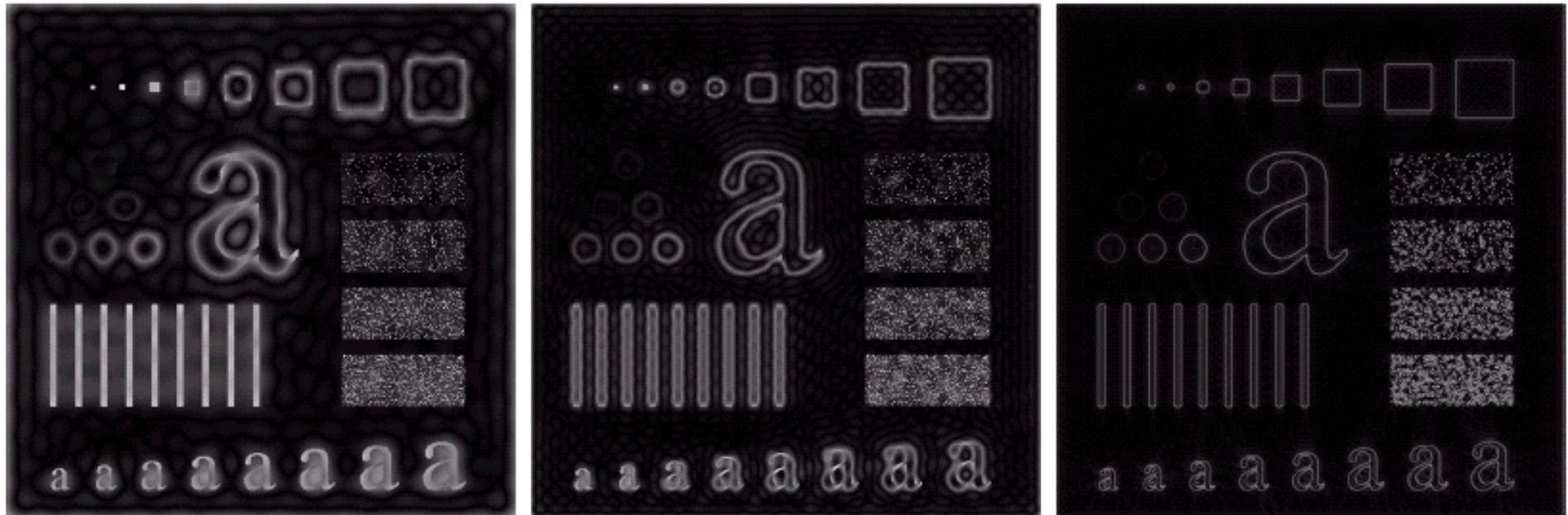
prb@2007

Imágenes: Gonzalez&Wood

21

**FIGURE 4.22** Top row: Perspective plot, image representation, and cross section of a typical ideal highpass filter. Middle and bottom rows: The same sequence for typical Butterworth and Gaussian highpass filters.

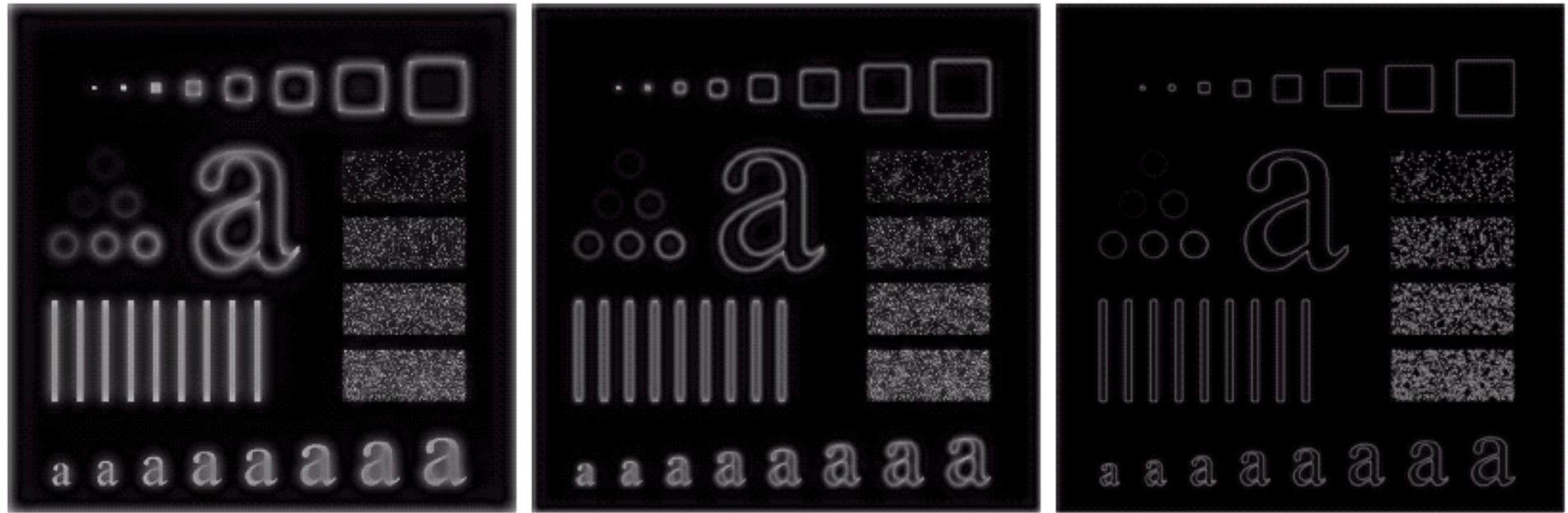
# Tratamiento de Imágenes: Dominio Frecuencias: “Filtro HP Ideal”



a b c

**FIGURE 4.24** Results of ideal highpass filtering the image in Fig. 4.11(a) with  $D_0 = 15, 30$ , and  $80$ , respectively. Problems with ringing are quite evident in (a) and (b).

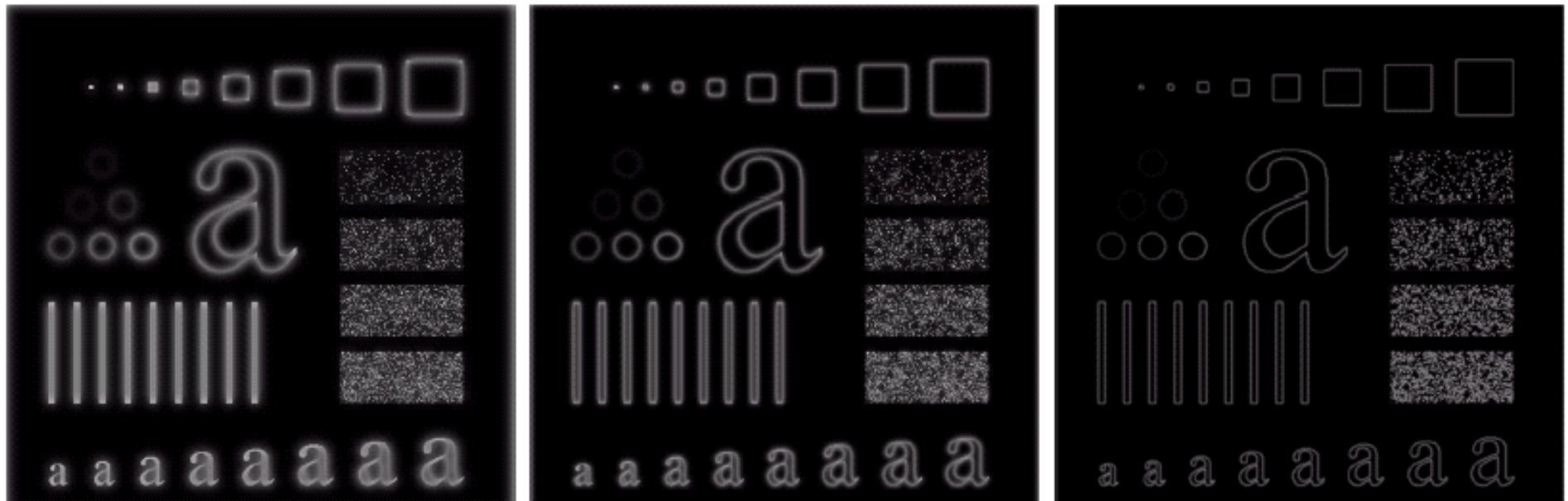
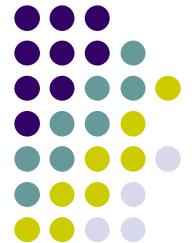
# Tratamiento de Imágenes: Dominio Frecuencias: “Filtro HP Butterworth”



a b | c

**FIGURE 4.25** Results of highpass filtering the image in Fig. 4.11(a) using a BHPF of order 2 with  $D_0 = 15$ , 30, and 80, respectively. These results are much smoother than those obtained with an ILPF.

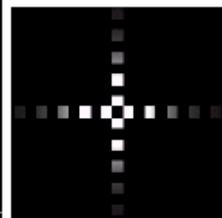
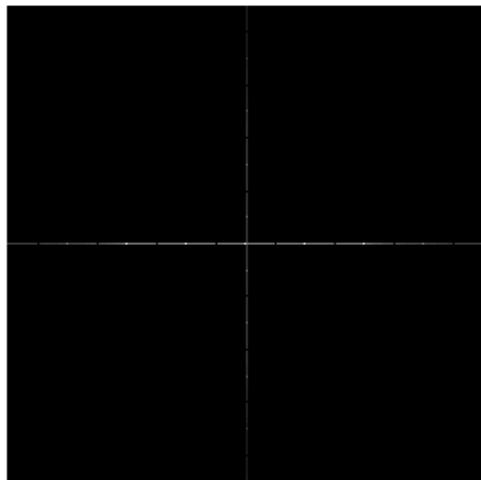
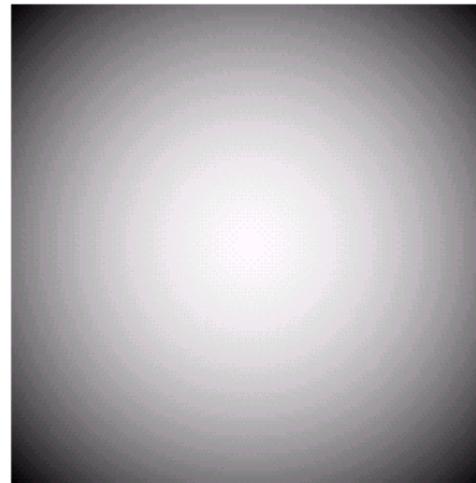
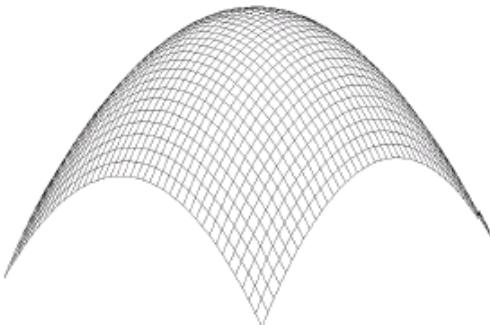
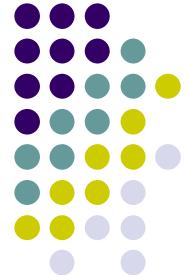
# Tratamiento de Imágenes: Dominio Frecuencias: “Filtro HP Gaussiano”



a b c

**FIGURE 4.26** Results of highpass filtering the image of Fig. 4.11(a) using a GHPF of order 2 with  $D_0 = 15$ , 30, and 80, respectively. Compare with Figs. 4.24 and 4.25.

# Tratamiento de Imágenes: Dominio Frecuencias: “Laplaciano en Frecuencia”



0	1	0
1	-4	1
0	1	0

a  
b  
c  
d  
e  
f

prb@2007

Imágenes: Gonzalez&Wood

**FIGURE 4.27** (a) 3-D plot of Laplacian in the frequency domain. (b) Image representation of (a). (c) Laplacian in the spatial domain obtained from the inverse DFT of (b). (d) Zoomed section of the origin of (c). (e) Gray-level profile through the center of (d). (f) Laplacian mask used in Section 3.7.

# Tratamiento de Imágenes: Dominio Frecuencias:



a b  
c d

**FIGURE 4.28**

- (a) Image of the North Pole of the moon.  
(b) Laplacian filtered image.  
(c) Laplacian image scaled.  
(d) Image enhanced by using Eq. (4.4-12).  
(Original image courtesy of NASA.)



# Tratamiento de Imágenes: Dominio Frecuencias: Aplicación



a b  
c d

**FIGURE 4.30**  
(a) A chest X-ray image. (b) Result of Butterworth highpass filtering. (c) Result of high-frequency emphasis filtering. (d) Result of performing histogram equalization on (c). (Original image courtesy Dr. Thomas R. Gest, Division of Anatomical Sciences, University of Michigan Medical School.)

