LECTURE 18: PLANAR ARRAYS, CIRCULAR ARRAYS

1. Planar arrays

Planar arrays are more versatile; they provide more symmetrical patterns with lower side lobes, much higher directivity (narrow main beam). They can be used to scan the main beam toward any point in space.

Applications – tracking radars, remote sensing, communications, etc.

1.1 The array factor of a rectangular planar array

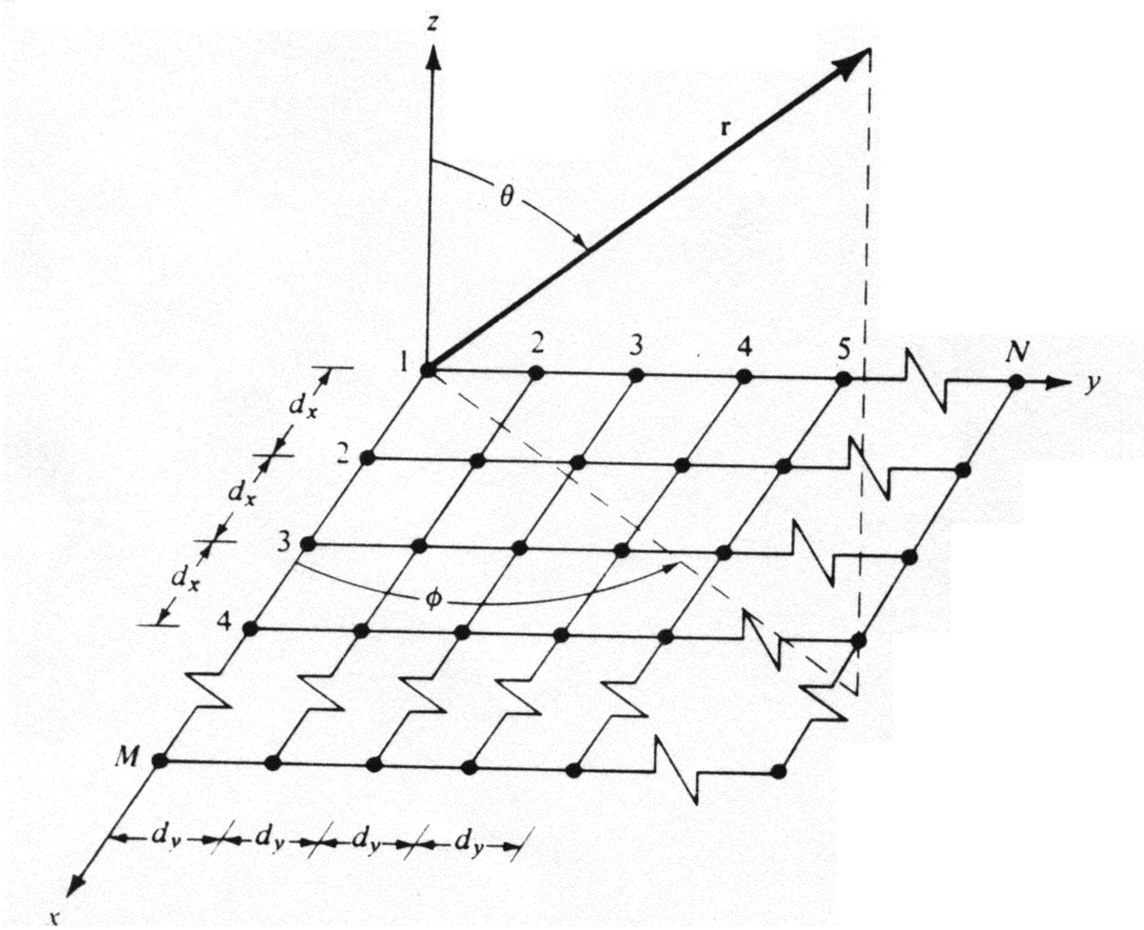


Fig. 6.23(b), pp.310, Balanis

The AF of a linear array of M elements along the x-axis is:

$$AF_{x1} = \sum_{m=1}^{M} I_{m1} e^{j(m-1)(kd\sin\theta\cos\phi+\beta_x)} \tag{18.1}$$

where $\sin\theta\cos\phi = \cos\gamma_x$ is the directional cosine with respect to the x-axis. It is assumed that all elements are equispaced with an interval of d_x and a progressive shift β_x. I_{m1} denotes the excitation amplitude of the element at the point with coordinates: $x=(m-1)d_x$, $y=0$. In the figure above, this is the element of the m-th row and the 1st column of the array matrix.

If N such arrays are placed next to each other in the y direction, a rectangular array will be formed. We shall assume again that they are equispaced at a distance of d_y and there is a progressive phase shift along each row of β_y. It will be also assumed that the normalized current distribution along each of the x-directed array is the same but the absolute values correspond to a factor of I_{1n} $(n=1,...,N)$. Then, the AF of the entire array will be:

$$AF = \sum_{n=1}^{N} I_{1n} \left[\sum_{m=1}^{N} I_{m1} e^{j(m-1)(kd_x\sin\theta\cos\phi+\beta_x)} \right] e^{j(n-1)(kd_y\sin\theta\cos\phi+\beta_y)} \tag{18.2}$$

or

$$AF = S_{x_M} \cdot S_{y_N}, \tag{18.3}$$

where:

$$S_{x_M} = AF_{x1} = \sum_{m=1}^{M} I_{m1} e^{j(m-1)(kd\sin\theta\cos\phi+\beta_x)}, \text{ and}$$

$$S_{y_N} = AF_{1y} = \sum_{n=1}^{N} I_{1n} e^{j(n-1)(kd_y\sin\theta\sin\phi+\beta_y)}$$

In the array factors above:

$$\begin{aligned} \sin\theta\cos\phi &= \hat{x}\cdot\hat{r} = \cos\gamma_x \\ \sin\theta\sin\phi &= \hat{y}\cdot\hat{r} = \cos\gamma_y \end{aligned} \tag{18.4}$$

The pattern of a rectangular array is the product of the array factors of the linear arrays in the x and y directions.

For a uniform planar (rectangular) array $I_{m1} = I_{1n} = I_0$, for all m and n, i.e., all elements have the same excitation amplitudes.

$$AF = I_0 \sum_{n=1}^{N} e^{j(m-1)(kd_x \sin\theta \cos\phi + \beta_x)} \sum_{n=1}^{N} e^{j(n-1)(kd_y \sin\theta \sin\phi + \beta_y)} \quad (18.5)$$

The normalized array factor can be obtained as:

$$\boxed{AF_n(\theta,\phi) = \left\{ \frac{1}{M} \frac{\sin\left(M \frac{\psi_x}{2}\right)}{\sin\left(\frac{\psi_x}{2}\right)} \right\} \left\{ \frac{1}{N} \frac{\sin\left(N \frac{\psi_y}{2}\right)}{\sin\left(\frac{\psi_y}{2}\right)} \right\}}, \quad (18.6)$$

where:

$$\psi_x = kd_x \sin\theta \cos\phi + \beta_x$$

$$\psi_y = kd_y \sin\theta \sin\phi + \beta_y$$

The major lobe (principal maximum) and grating lobes of the terms:

$$S_{x_M} = \frac{1}{M} \frac{\sin\left(M \frac{\psi_x}{2}\right)}{\sin\left(\frac{\psi_x}{2}\right)} \quad (18.7)$$

$$S_{y_N} = \frac{1}{N} \frac{\sin\left(N \frac{\psi_y}{2}\right)}{\sin\left(\frac{\psi_y}{2}\right)} \quad (18.8)$$

are located at angles such that:

$$kd_x \sin\theta_m \cos\phi_m + \beta_x = \pm 2m\pi, \quad m = 0,1,\ldots \quad (18.9)$$

$$kd_y \sin\theta_n \sin\phi_n + \beta_y = \pm 2n\pi, \quad n = 0,1,\ldots \quad (18.10)$$

The principal maxima correspond to $m = 0, \; n = 0$.

In general, β_x and β_y are independent from each other. But, if it is required that the main beams of S_{x_M} and S_{y_N} intersect (which is usually the case), then the common main beam is in the direction:

$$\theta = \theta_0 \text{ and } \phi = \phi_0, \; m = n = 0 \tag{18.11}$$

If the principal maximum is specified by (θ_0, ϕ_0), then the progressive phases β_x and β_y must satisfy:

$$\beta_x = -kd_x \sin\theta_0 \cos\phi_0 \tag{18.12}$$

$$\beta_y = -kd_y \sin\theta_0 \sin\phi_0 \tag{18.13}$$

When β_x and β_y are specified, the direction of the main beam can be found by simultaneously solving (18.12) and (18.13):

$$\tan\phi_0 = \frac{\beta_y d_x}{\beta_x d_y} \tag{18.14}$$

$$\sin\theta_0 = \pm\sqrt{\left(\frac{\beta_x}{kd_x}\right)^2 + \left(\frac{\beta_y}{kd_y}\right)^2} \tag{18.15}$$

The grating lobes can be located by substituting (18.12) and (18.13) in (18.9) and (18.10):

$$\tan\phi_{mn} = \frac{\sin\theta_0 \sin\phi_0 \pm n\lambda/d_y}{\sin\theta_0 \cos\phi_0 \pm m\lambda/d_x} \tag{18.16}$$

$$\sin\theta_{mn} = \frac{\sin\theta_0 \cos\phi_0 \pm m\lambda/d_x}{\cos\phi_{mn}} = \frac{\sin\theta_0 \sin\phi_0 \pm n\lambda/d_y}{\sin\phi_{mn}} \tag{18.17}$$

To avoid grating lobes, the spacing between the elements must be less than λ ($d_y < \lambda$ and $d_y < \lambda$). In order a true grating lobe to occur, both equations (18.16) and (18.17) must have a real solution (θ_{mn}, ϕ_{mn}).

3-D pattern of a 5-element square planar uniform array without grating lobes ($d = \lambda/4$, $\beta_x = \beta_y = 0$):

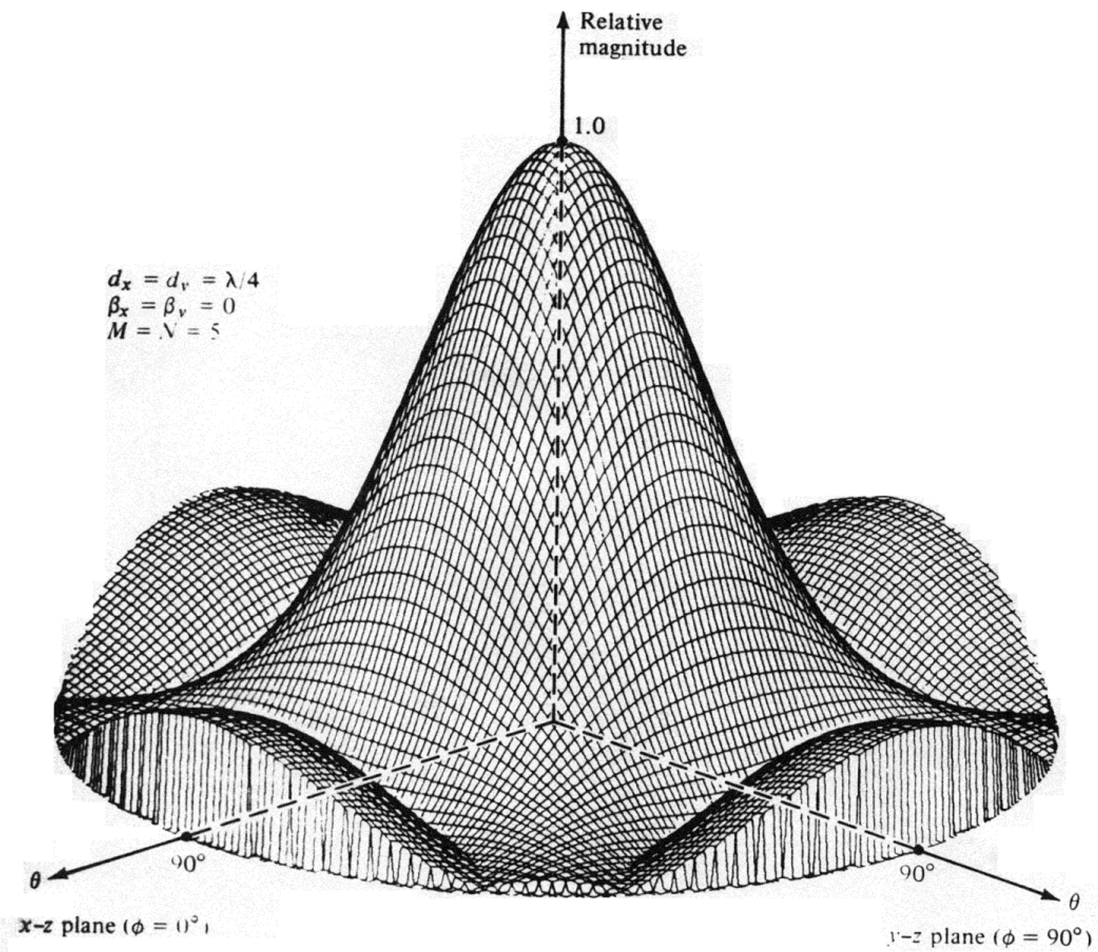


Fig. 6.24, pp.313 Balanis

3-D pattern of a 5-element square planar uniform array without grating lobes ($d = \lambda/2$, $\beta_x = \beta_y = 0$):

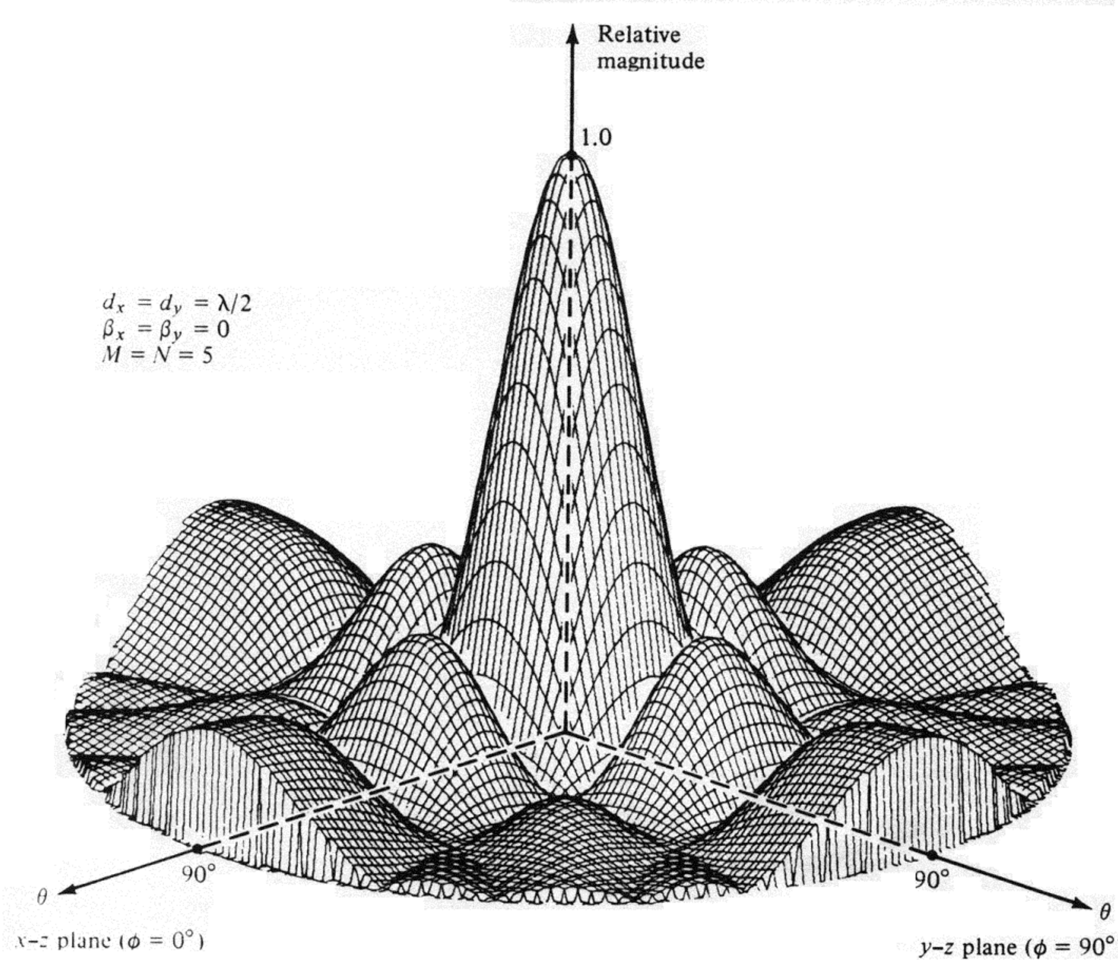
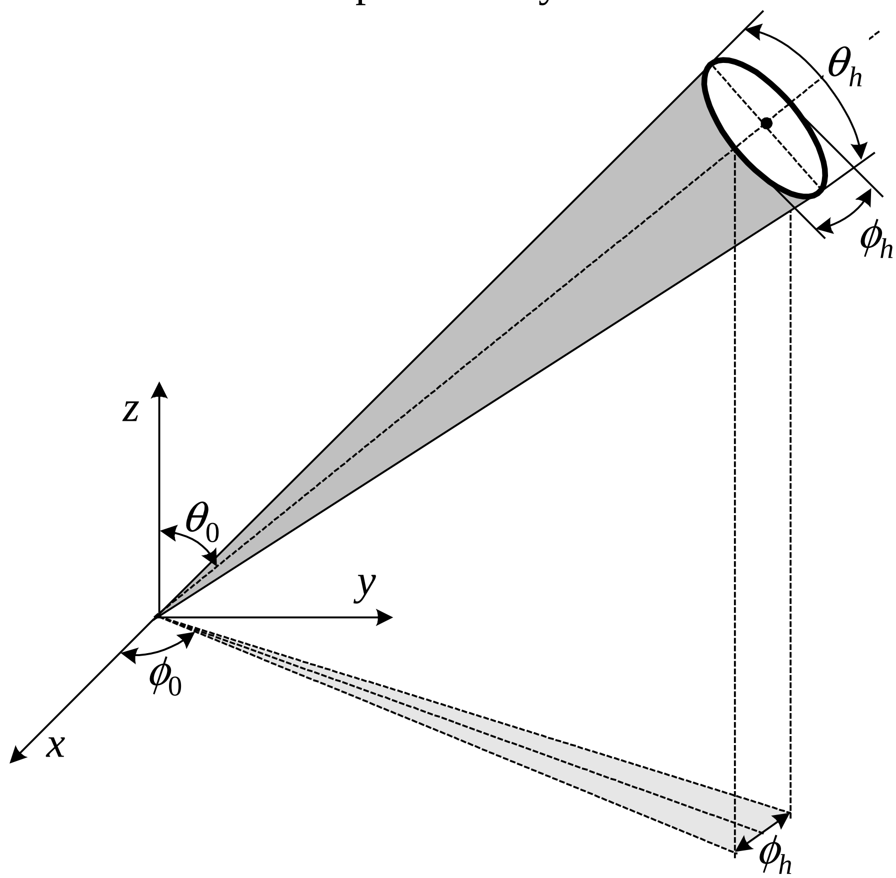


Fig. 6.25, pp.314, Balanis

Notice the considerable decrease in the beamwidth as the spacing is increased from $\lambda/4$ to $\lambda/2$.

1.2 The beamwidth of a planar array



A simple procedure, proposed by R.S. Elliot[1] will be outlined. It is based on the use of the beamwidths of the linear arrays building the planar array.

For a large array, whose maximum is near the broad side, the elevation plane HPBW is approximately:

$$\theta_h = \sqrt{\frac{1}{\cos^2\theta_0\left[\vartriangle\theta_x^{-2}\cos^2\phi_0 + \vartriangle\theta_y^{-2}\sin^2\phi_0\right]}} \qquad (18.18)$$

[1] "Beamwidth and directivity of large scanning arrays", *The Microwave Journal*, Jan. 1964, pp.74-82

where: (θ_0, ϕ_0) specify the main-beam direction;

$\vartriangle\theta_x$ is the HPBW of a linear broadside array whose number of elements M and amplitude distribution is the same as that of the x-axis linear arrays building the planar array;

$\vartriangle\theta_y$ is the HPBW of a linear BSA whose number of elements N and amplitude distribution is the same as those of the y-axis linear arrays building the planar array.

The HPBW in the plane, which is orthogonal to the $\phi = \phi_0$ plane and contains the maximum, is:

$$\phi_h = \sqrt{\frac{1}{\vartriangle\theta_x^{-2} \sin^2 \phi_0 + \vartriangle\theta_y^{-2} \cos^2 \phi_0}} \tag{18.19}$$

For a square array $(M = N)$ with amplitude distributions along the x and y axes of the same type, equations (18.18) and (18.19) reduce to:

$$\theta_h = \frac{\vartriangle\theta_x}{\cos\theta_0} = \frac{\vartriangle\theta_y}{\cos\theta_0} \tag{18.20}$$

$$\phi_h = \vartriangle\theta_x = \vartriangle\theta_y \tag{18.21}$$

From (18.20), it is obvious that the HPBW in the elevation plane very much depends on the elevation angle θ_0 of the main beam. The HPBW in the azimuthal plane ϕ_h does not depend on the elevation angle θ_0.

The beam solid angle of the planar array can be approximated by:

$$\Omega_A = \theta_h \phi_h \tag{18.22}$$

or

$$\Omega_A = \frac{\triangle\theta_x \triangle\theta_y}{\cos^2\theta_0 \sqrt{\left[\sin^2\phi_0 + \frac{\triangle\theta_y^2}{\triangle\theta_x^2}\cos^2\phi_0\right]\left[\sin^2\phi_0 + \frac{\triangle\theta_x^2}{\triangle\theta_y^2}\cos^2\phi_0\right]}} \tag{18.23}$$

1.3 Directivity

The general expression for the calculation of the directivity of an array is:

$$D_0 = 4\pi \frac{|AF(\theta_0,\phi_0)|^2}{\int_0^{2\pi}\int_0^{\pi} |AF(\theta_0,\phi_0)|^2 \sin\theta d\theta d\phi} \tag{18.24}$$

For large planar arrays, which are nearly broadside, (18.24) reduces to:

$$D_0 = \pi D_x D_y \cos\theta_0 \tag{18.25}$$

where D_x is the directivity of the respective linear BSA, *x*-axis;
D_y is the directivity of the respective linear BSA, *y*-axis.

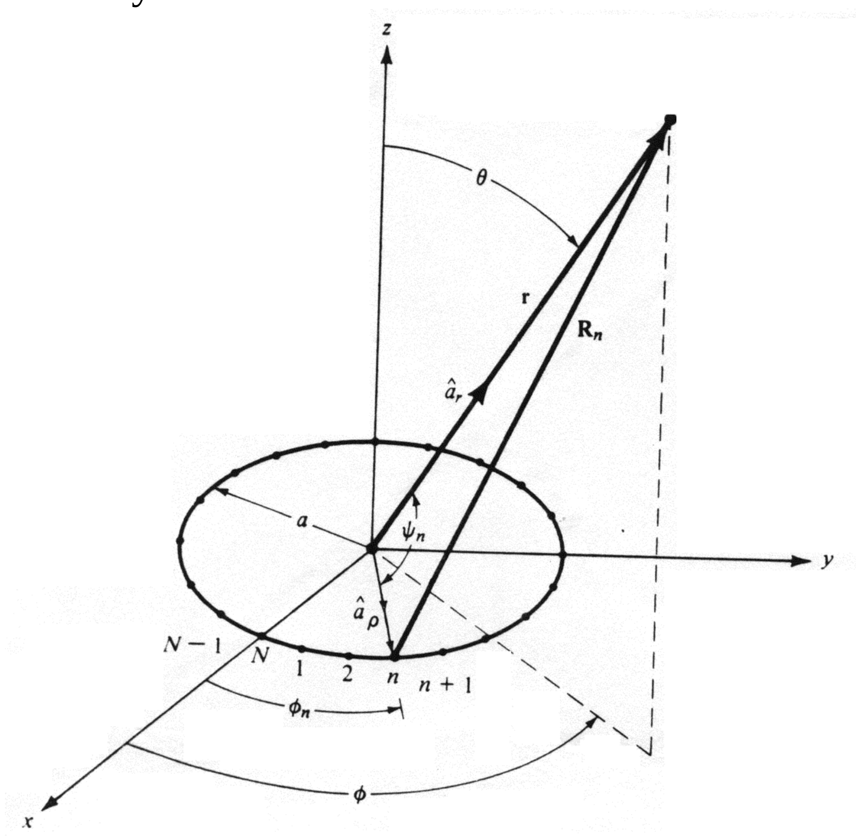
One can also use the array solid beam angle Ω_A in (18.23) to calculate the approximate directivity of a nearly broadside planar array:

$$D_0 \simeq \frac{\pi^2}{\Omega_{A[Sr]}} \simeq \frac{32400}{\Omega_{A[\deg^2]}} \tag{18.26}$$

Remember:

1) The main beam direction is controlled through the phase shifts, β_x and β_y.
2) The beamwidth and side-lobe levels are controlled through the amplitude distribution.

2. Circular array



2.1 Array factor

The normalized field can be written as:

$$E(r,\theta,\phi)=\sum_{n=1}^{N} a_n \frac{e^{-jkR_n}}{R_n} \qquad (18.27)$$

where:

$$R_n=\sqrt{r^2+a^2-2ar\cos\psi_n} \qquad (18.28)$$

For $r \gg a$, (18.28) reduces to:

$$R_n \simeq r-a\cos\psi_n \simeq r-a\left(\hat{a}_{\rho_n}\cdot\hat{r}\right) \qquad (18.29)$$

In rectangular coordinate system:

$$\left|\begin{array}{l}\hat{a}_{\rho_n}=\hat{x}\cos\phi_n+\hat{y}\sin\phi_n\\ \hat{r}=\hat{x}\sin\theta\cos\phi+\hat{y}\sin\theta\sin\phi+\hat{z}\cos\theta\end{array}\right.$$

Therefore:

$$R_n = r - a\sin\theta\left(\cos\phi_n\cos\phi + \sin\phi_n\sin\phi\right) \qquad (18.30)$$

Finally, R_n is approximated in the phase terms as:

$$R_n = r - a\sin\theta\cos\left(\phi - \phi_n\right) \qquad (18.31)$$

For the amplitude term, the approximation

$$\frac{1}{R_n} \simeq \frac{1}{r}, \text{ all } n \qquad (18.32)$$

is made.

Assuming the approximations (18.31) and (18.32) are valid, the far-zone array field is reduced to:

$$E\left(r,\theta,\phi\right) = \frac{e^{-jkr}}{r}\sum_{n=1}^{N} a_n e^{jka\sin\theta\cos\left(\phi-\phi_n\right)} \qquad (18.33)$$

where: a_n is the excitation coefficient (amplitude and phase);

$\phi_n = \frac{2\pi}{N}n$ is the angular position of the *n*-th element.

In general, the excitation coefficient can be represented as:

$$a = I_n e^{j\alpha_n}, \qquad (18.34)$$

where I_n is the amplitude term, and α_n is the phase of the excitation of the *n*-th element relative to a chosen array element of zero phase.

$$\Rightarrow E\left(r,\theta,\phi\right) = \frac{e^{-jkr}}{r}\sum_{n=1}^{N} I_n e^{j\left[ka\sin\theta\cos\left(\phi-\phi_n\right)+\alpha_n\right]} \qquad (18.35)$$

The AF is obtained as:

$$AF\left(\theta,\phi\right) = \sum_{n=1}^{N} I_n e^{j\left[ka\sin\theta\cos\left(\phi-\phi_n\right)+\alpha_n\right]} \qquad (18.36)$$

Expression (18.36) represents the AF of a circular array of N equispaced elements. The maximum of the AF occurs when all the phase terms in (18.36) equal unity, or:

$$ka\sin\theta\cos\left(\phi-\phi_n\right)+\alpha_n = 2m\pi, \quad m = 0, \pm1, \pm2, \text{ all } n \qquad (18.37)$$

The principal maximum ($m=0$) is defined by the direction (θ_0,ϕ_0), for which:

$$\alpha_n = -ka\sin\theta_0\cos\left(\phi_0-\phi_n\right),\quad n=1,2,...,N \tag{18.38}$$

If a circular array is required to have maximum radiation in the direction (θ_0,ϕ_0), then the phases of its excitations will have to fulfil (18.38). The AF of such an array is:

$$AF\left(\theta,\phi\right)=\sum_{n=1}^{N} I_n e^{jka\left[\sin\theta\cos(\phi-\phi_n)-\sin\theta_0\cos(\phi_0-\phi_n)\right]} \tag{18.39}$$

$$AF(\theta,\phi)=\sum_{n=1}^{N} I_n e^{jka(\cos\psi_n-\cos\psi_{0n})} \tag{18.40}$$

Here:

$\psi_n=\cos^{-1}\left[\sin\theta\cos(\phi-\phi_n)\right]$ is the angle between $\hat{r}$ and $\hat{a}_{\rho_n}$;

$\psi_{0_n}=\cos^{-1}\left[\sin\theta_0\cos(\phi_0-\phi_n)\right]$ is the angle between $\hat{a}_{\rho_n}$ and $\hat{r}_{\max}$ pointing in the direction of maximum radiation.

As the radius of the array a becomes very large as compared to λ, the directivity of the uniform circular array ($I_n=I_0$, all n) approaches the value of N.

Uniform circular array 3-D pattern (N=10, $ka = \frac{2\pi}{\lambda} a = 10$)