Friis Transmission Equation and the Radar Range Equation

1. Other antenna equivalent areas

Before, we have defined the antenna effective area (or effective aperture) as the area, which when multiplied by the incident wave power density, produces the power delivered to the load (the terminals of the antenna) P_A. In a very similar manner, one can define the ***antenna scattering area*** A_s. It is the area, which when multiplied with the incident wave power density, produces the re-radiated (scattered) power.

$$A_s = \frac{P_s}{W_i} = \frac{|I_A|^2 R_r / 2}{W_i}, \text{ m}^2 \tag{6.1}$$

In the case of conjugate matching:

$$A_s = \frac{|V_A|^2}{8W_i} \frac{R_r}{(R_r + R_l)^2} = \frac{|V_A|^2}{8W_i} \frac{R_r}{R_A^2}, \text{ m}^2 \tag{6.2}$$

The ***loss area*** is the area, which when multiplied by the incident wave power density, produces the dissipated (as heat) power of the antenna.

$$A_l = \frac{P_l}{W_i} = \frac{|I_A|^2 R_l / 2}{W_i}, \text{ m}^2 \tag{6.3}$$

In the case of conjugate matching:

$$A_l = \frac{|V_A|^2}{8W_i} \frac{R_l}{(R_r + R_l)^2} = \frac{|V_A|^2}{8W_i} \frac{R_l}{R_A^2}, \text{ m}^2 \tag{6.4}$$

The capture area is the area, which when multiplied with the incident wave power density, produces the total power intercepted by the antenna.

$$A_c = \frac{P_t}{W_i} = \frac{|I_A|^2 (R_r + R_l + R_L) / 2}{W_i} \tag{6.5}$$

In the case of conjugate matching:

$$A_c = \frac{|V_A|^2}{8W_i} \frac{(R_r + R_l + R_L)}{(R_r + R_l)^2} = \frac{|V_A|^2}{8W_i} \frac{(R_A + R_L)}{R_A^2} = \frac{|V_A|^2}{4W_i} \frac{1}{R_A} \tag{6.6}$$

Always, the capture area is the sum of the effective area, the loss area and the scattering area:

$$A_c = A_e + A_l + A_s \tag{6.7}$$

When conjugate matching is achieved:

$$A_e = A_l + A_s = \frac{1}{2} A_c \tag{6.8}$$

If conjugate matching is achieved for a loss-free antenna, then

$$A_e = A_s = \frac{1}{2} A_c \tag{6.9}$$

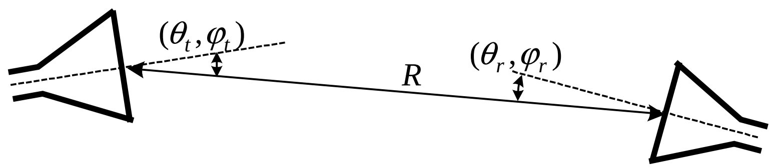
2. Friis transmission equation

The analysis and design of wireless communication systems requires the use of the Friis transmission equation. It relates the power, which is fed to the transmitting antenna, to the power, which is received by the receiving antenna, when the two antennas are separated by a large enough distance ($R >> 2D_{\max}^2/\lambda$). This means that the two antennas are positioned in each other's far zones. Friis transmission equation will be derived below.

A transmitting antenna produces power density $W_t(\theta_t,\varphi_t)$ in the direction (θ_t,φ_t). This power density depends on the transmitting antenna gain in the given direction $G(\theta_t,\varphi_t)$, on the power of the transmitter P_t fed to it, and on the distance R between the antenna and the observation point as:

$$W_t = \frac{P_t}{4\pi R^2} G_t(\theta_t,\varphi_t) = \frac{P_t}{4\pi R^2} e_t D_t(\theta_t,\varphi_t) \tag{6.10}$$

Here, e_t denotes the radiation efficiency of the transmitting antenna, and D_t is its directivity. Assume that R is exactly the distance between the transmitting and the receiving antennas. Then, (6.10) gives the power density produced by the transmitting antenna in the location of the receiving antenna.



The power at the terminals of the receiving antenna can be expressed via its effective area A_r and W_t:

$$P_r = A_r W_t \tag{6.11}$$

To include polarization losses and heat losses in the receiving antenna, one should add the radiation efficiency factor of the receiving antenna e_r and the PLF:

$$P_r = e_r \text{PLF} \cdot A_r W_t = A_r W_t e_r \, |\hat{\rho}_t \cdot \hat{\rho}_r|^2 \tag{6.12}$$

$$\Rightarrow P_r = \underbrace{D_r(\theta_r,\varphi_r)\frac{\lambda^2}{4\pi}}_{A_r} \cdot W_t e_r \, |\hat{\rho}_t \cdot \hat{\rho}_r|^2 \tag{6.13}$$

Here, e_r is the radiation efficiency of the receiving antenna, and D_r is its directivity. The signal is incident onto the receiving antenna from a direction (θ_r,φ_r), which is defined in the coordinate system of the receiving antenna.

$$\Rightarrow P_r = D_r(\theta_r,\varphi_r)\frac{\lambda^2}{4\pi} \cdot \underbrace{\frac{P_t}{4\pi R^2} e_t D_t(\theta_t,\varphi_t)}_{W_t} \cdot e_r \, |\hat{\rho}_t \cdot \hat{\rho}_r|^2 \tag{6.14}$$

The ratio of the received to the transmitted power is obtained as:

$$\frac{P_r}{P_t} = e_t e_r |\hat{\rho}_t \cdot \hat{\rho}_r|^2 \left(\frac{\lambda}{4\pi R}\right)^2 D_t(\theta_t, \varphi_t) D_r(\theta_r, \varphi_r) \quad (6.15)$$

If the impedance-mismatch loss factor is to be included in both, the receiving and the transmitting antenna systems, the above ratio has to be given as:

$$\frac{P_r}{P_t} = (1 - |\Gamma_t|^2)(1 - |\Gamma_r|^2) e_t e_r |\hat{\rho}_t \cdot \hat{\rho}_r|^2 \left(\frac{\lambda}{4\pi R}\right)^2 D_t(\theta_t, \varphi_t) D_r(\theta_r, \varphi_r) \quad (6.16)$$

The above equations are variations of Friis transmission equation, which is well known in the theory of EM wave propagation.

For the case of impedance matched and polarization matched transmitting and receiving antennas, Friis equation becomes:

$$\frac{P_r}{P_t} = \left(\frac{\lambda}{4\pi R}\right)^2 D_t(\theta_t, \varphi_t) D_r(\theta_r, \varphi_r) \quad (6.17)$$

The factor $(\lambda / 4\pi R)^2$ is called the ***free-space loss factor***. It reflects the decrease in the power density due to the spherical spread of the EM wave. Notice that the free-space loss factor decreases for shorter wavelengths (i.e. for higher frequencies). This is not a propagation effect but is rather due to the increased effective apertures of the antennas for shorter wavelengths.

Friis transmission equation is frequently used to calculate the maximum range at which a wireless link can operate. For that, one needs to know the nominal power of the transmitter P_t, all the parameters of the transmitting and receiving antenna systems (such as polarization, gain, losses, impedance mismatch), and the minimum power at which the receiver can operate reliably $P_{r\,\min}$. Then,

$$R_{\max}^2 = (1 - |\Gamma_t|^2)(1 - |\Gamma_r|^2) e_t e_r |\hat{\rho}_t \cdot \hat{\rho}_r|^2 \left(\frac{\lambda}{4\pi}\right)^2 \left(\frac{P_t}{P_{r\,\min}}\right) D_t(\theta_t, \varphi_t) D_r(\theta_r, \varphi_r) \quad (6.18)$$

The minimum power at which the receiver can operate reliably is dependent on numerous factors, of which very important is the signal to noise ratio (SNR). There are different sources of noise but we will be mostly concerned with the noise of the antenna itself. This topic will be considered in the next module.

3. Radar range equation

Radar cross-section (RCS) or echo area

The RCS is a far-field characteristic of radar targets, which create an echo far field by scattering (reflecting) the radar EM wave. The RCS of a target σ is the equivalent area intercepting that amount of power, which when scattered isotropically produces at the receiver a density, which is equal to that scattered by the target itself:

$$\sigma = \lim_{R\to\infty}\left[4\pi R^2 \frac{W_s}{W_i}\right] = \lim_{R\to\infty}\left[4\pi R^2 \frac{|\vec{E}_s|^2}{|\vec{E}_i|^2}\right], \text{ m}^2 \tag{6.19}$$

Here:

R is the distance from the target, m;
W_i is the incident power density, W/m^2;
W_s is the scattered power density, W/m^2.

To understand better the above definition, one can re-write it in an equivalent form as:

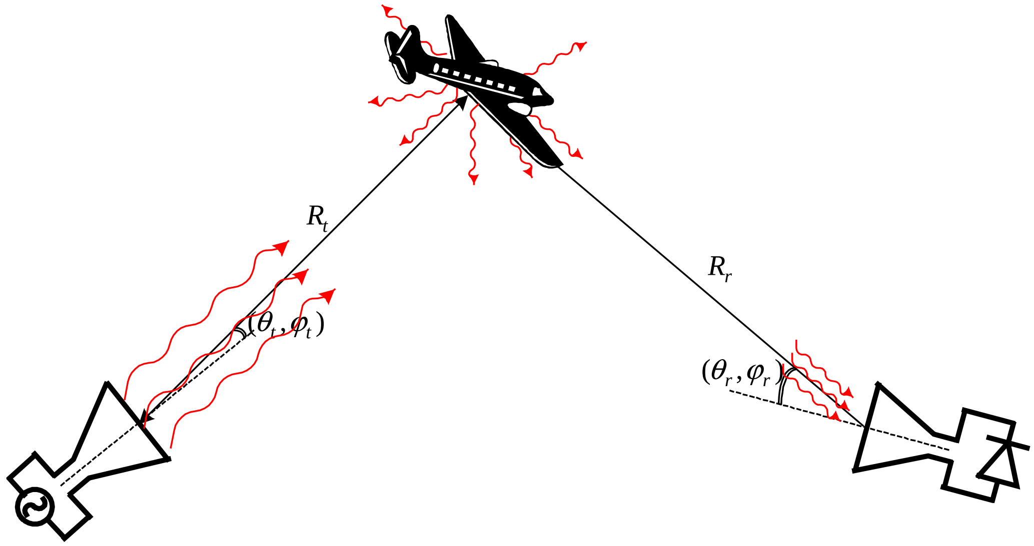
$$\lim_{R\to\infty}\left[\frac{\sigma W_i(R)}{4\pi R^2}\right] = W_s \tag{6.20}$$

The product σW_i represents some fictitious intercepted power, which is then scattered (re-radiated) isotropically to create a fictitious spherical wave, whose power density in the far zone decreases with distance as $1/R^2$. It is then expected that the above limit is finite and produces certain scattered power density. This scattered power density must be equal to the real scattered power density W_s produced by the real scatterer (the radar target).

It should be noted that the RCS has little in common with any of the cross-sections of the actual scatterer. However, it is very representative of the reflection properties of the target. It very much depends on the angle of incidence, on the angle of observation, on the shape of the scatterer, on the EM properties of the matter that it is built of, and on the wavelength. The RCS of targets is similar to the concept of effective aperture of antennas.

The radar range equation (RRE) gives the ratio of the transmitted power (fed to the transmitting antenna) to the received power after it has been scattered (re-radiated) by a target of cross-section σ.

In the general radar scattering problem, there is a transmitting and a receiving antenna, and they may be located in different positions as it is shown in figure below. This is called *bistatic scattering*. Often, one antenna is used to transmit an EM pulse and to receive the echo from the target. This case is referred to as *monostatic scattering*, or *backscattering*. Bear in mind that the RCS of a target may considerably differ as the location of the transmitting and receiving antennas change.



We will now derive the radar range equation. Assume that the power density of the transmitted wave at the target location is W_t, where

$$W_t = \frac{P_t G_t(\theta_t,\varphi_t)}{4\pi R_t^2} = \frac{P_t e_t D_t(\theta_t,\varphi_t)}{4\pi R_t^2}, \text{ W/m}^2 \qquad (6.21)$$

The target is represented by its RCS σ, which is used to calculate the captured power $P_c = \sigma W_t$ (W), which when scattered isotropically will give the power density at the receiving antenna that is actually created by the target. The density of the re-radiated (scattered) power at the receiving antenna is:

$$W_r = \frac{P_c}{4\pi R_r^2} = \frac{\sigma W_t}{4\pi R_r^2} = e_t \sigma \frac{P_t D_t(\theta_t,\varphi_t)}{(4\pi R_t R_r)^2} \qquad (6.22)$$

The power transferred to the receiver is:

$$P_r = e_r A_r W_r = e_r \cdot \left(\frac{\lambda^2}{4\pi}\right) D_r(\theta_r,\varphi_r) \cdot e_t \sigma \frac{P_t D_t(\theta_t,\varphi_t)}{(4\pi R_t R_r)^2} \qquad (6.23)$$

Re-arranging and including impedance mismatch losses as well as polarization losses, yields the complete radar range equation:

$$\frac{P_r}{P_t} = e_t e_r (1-|\Gamma_t|^2)(1-|\Gamma_r|^2)|\hat{\rho}_t \cdot \hat{\rho}_r|^2 \sigma \left(\frac{\lambda}{4\pi R_t R_r}\right)^2 \frac{D_t(\theta_t,\varphi_t) D_r(\theta_r,\varphi_r)}{4\pi} \qquad (6.24)$$

For polarization matched loss-free antennas aligned for maximum directional radiation and reception:

$$\frac{P_r}{P_t} = \sigma \left(\frac{\lambda}{4\pi R_t R_r}\right)^2 \frac{D_t(\theta_t,\varphi_t) D_r(\theta_r,\varphi_r)}{4\pi} \qquad (6.25)$$

The radar range equation is often used to calculate the maximum range of a radar system. As in the case of Friis transmission equation, one needs to know all the parameters of both, the transmitting and the receiving antennas, as well as the minimum receive power at which the receiver operates reliably. Then,

$$(R_t R_r)^2 = e_t e_r (1-|\Gamma_t|^2)(1-|\Gamma_r|^2)\,|\hat{\rho}_t \cdot \hat{\rho}_r|^2 \,\frac{P_t}{P_r}\sigma\left(\frac{\lambda}{4\pi}\right)^2 \frac{D_t(\theta_t,\varphi_t)D_r(\theta_r,\varphi_r)}{4\pi} \quad (6.26)$$

Finally, it should be noted that the above information on RCS and radar range calculations are only basic information on radar systems and electromagnetic scattering. Both of these subjects represent huge research areas by themselves, and are not going to be considered in detail in this course.