Lecture 7: Antenna Noise Temperature and System Signal-to-Noise Ratio

1. Antenna temperature

The performance of a telecommunication system depends very much on the signal-to-noise ratio (SNR) at the receiver's input. The electronic circuitry of the receiver (amplifiers, mixers, etc.) has its own contribution to the noise generation. However, the antenna itself is a significant source of noise. The antenna noise can be divided into two types of noise according to its physical source:

- noise due to the loss resistance of the antenna itself; and
- noise, which the antenna picks up from the surrounding environment.

Any object whose temperature is above the absolute zero radiates EM energy. Thus, each antenna is surrounded by noise sources, which create noise power at the antenna terminals. Here, we will not be concerned with technological sources of noise, which are a subject of the electromagnetic interference science. We are also not concerned with intentional sources of electromagnetic interference. We are concerned with natural sources of EM noise, such as sky noise and ground noise.

The concept of antenna temperature is not only associated with the EM noise. The relation between the object's temperature and the power it can create at the antenna terminals is used in passive remote sensing (radiometry). A radiometer can create temperature images of objects. Typically, the remote object's temperature is measured by comparison with the noise due to background sources and the receiver itself.

Every object with a physical temperature above zero ($0^\circ \text{K} = -273^\circ \text{C}$) possesses heat energy. The noise power per unit bandwidth is proportional to the object's temperature and is given by the Nyquist's relation:

$$p_h = kT_P \text{, W/Hz} \tag{7.1}$$

where

T_P is the physical temperature of the object in K (Kelvin degrees); and

k is Boltzmann's constant (1.38×10^{-23} J/K).

Often, the heat energy is assumed evenly distributed in a certain frequency band $\triangle f$. Then, the associated heat power in the frequency band $\triangle f$ is:

$$P_h = kT_P \triangle f \text{, W} \tag{7.2}$$

Every object with a physical temperature above zero and possessing some finite conductivity radiates EM power. This power depends on the ability of the object's surface to let the heat leak out. This radiated heat power is associated with the so-called ***equivalent temperature*** or ***brightness temperature*** of the body via the power-temperature relation in (7.2):

$$P_B = kT_B \triangle f \text{, W} \tag{7.3}$$

The brightness temperature T_B is proportional to the physical temperature of the body T_P:

$$T_B = (1 - |\Gamma_s|^2) \cdot T_P = \varepsilon T_P \text{, K} \tag{7.4}$$

where:

Γ_s is the reflection coefficient of the surface of the body for the given polarization of the wave; and

ε is called the ***emissivity*** of the body.

The power radiated by the body P_B (the power associated with the brightness temperature), when intercepted by the antenna, creates power at the antenna terminals P_A. The equivalent temperature associated with the received power P_A at the antenna terminals is called ***antenna temperature*** T_A of the object, where again $P_A = kT_A \vartriangle f$.

The received power can be calculated if the antenna effective aperture A_e (m^2) is known and if the power density W_B (W/m^2) created by the bright body at the antenna's location is known:

$$P_A = A_e W_B \text{, W} \tag{7.5}$$

If the body radiates isotropically in all directions, then

$$W_B = \frac{P_B}{4\pi R^2} \text{, W/m}^2 \tag{7.6}$$

where

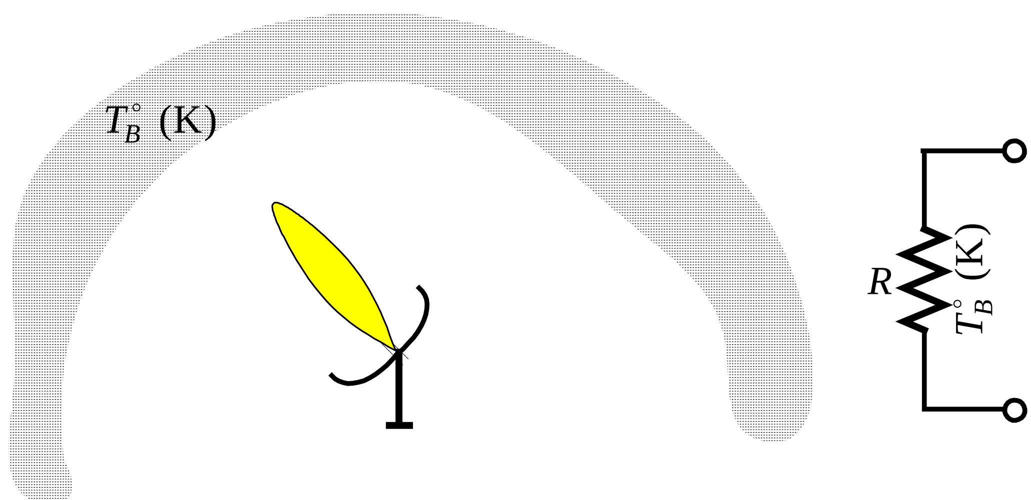
$P_B = kT_B \vartriangle f$ (W) is the brightness power radiated from the body,

R (m) is the distance between the object and the antenna.

Let us first assume that the entire antenna pattern (beam) "sees" an object of temperature T_B (K). We will assume that the antenna itself is lossless, i.e. it has no loss resistance, and, therefore, it does not generate noise itself. Then, certain noise power can be measured at its terminals as:

$$P_A = kT_B \triangle f \text{ , W} \tag{7.7}$$

This is the same power as the power that would be generated by a resistor set at temperature T_B (K).



The antenna temperature is related to the measured noise power as:

$$P_A = kT_A \triangle f \tag{7.8}$$

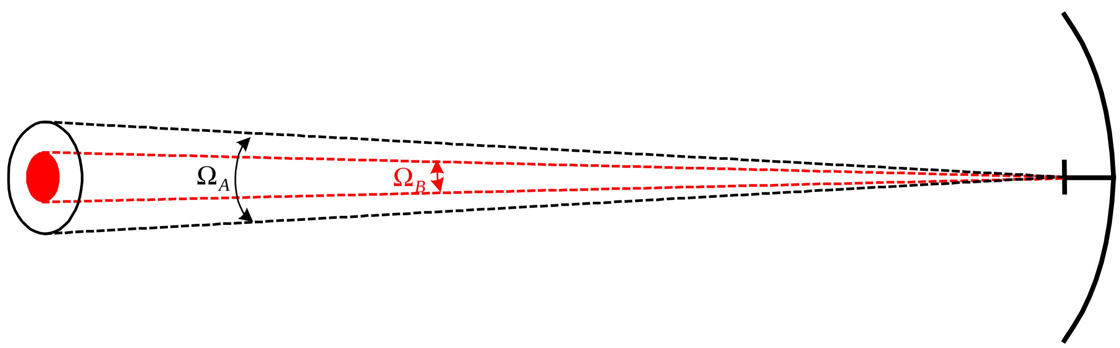
In the case described above (where the solid angle subtended by the noise source Ω_B is much larger than the antenna solid angle Ω_A), the antenna temperature T_A is exactly equal to the object's temperature T_B (if the antenna is loss-free):

$$T_A = T_B, \text{ if } \Omega_A \ll \Omega_B \tag{7.9}$$

The situation described above is of practical importance to the overall antenna noise performance. When an antenna is pointed right at the night sky, its noise temperature is very low: $T = 3^\circ$ to 5° K at frequencies between 1 and 10 GHz. This is exactly the noise temperature of the night sky. The higher the elevation angle, the less the sky temperature is. Sky noise is very much dependent on the frequency. It depends on the time of the day, too. It is due to cosmic rays (emanating from the sun, the moon and other bright sky objects), to atmospheric noise and also to man-made noise.

The noise temperature of ground is about $\approx 300^\circ$ K, and, of course, varies during the day. The noise temperature at approximately zero elevation angle (horizon) is about 100° to 150° K.

The opposite special case arises in radiometry and radioastronomy. The bright object (noise source) subtends such a small solid angle that it is well inside the antenna solid angle when the antenna is pointed at the object: $\Omega_B \ll \Omega_A$.



To separate the power received from the bright body from the background noise, the difference in the antenna temperature ΔT_A is measured with the beam on and off the object.

In the above case, the difference in the antenna temperature is not equal to the bright body temperature T_B, as it was in the case of a large noise object. However, both temperatures are proportional. The relation is derived below. The noise power intercepted by the antenna depends on the antenna effective aperture A_e and on the power density at the antenna's location created by the noise source W_B:

$$P_A = A_e W_B, \text{ W} \tag{7.10}$$

We will assume that the bright body radiates isotropically, and we will express the effective area by the antenna solid angle:

$$P_A = \frac{\lambda^2}{\Omega_A} \cdot \frac{P_B}{4\pi R^2}, \text{ W} \tag{7.11}$$

The distance between the noise source and the antenna R is related to the effective area of the body and the solid angle Ω_B it subtends as:

$$R^2 = \frac{S_B}{\Omega_B}, \text{ m}^2 \tag{7.12}$$

$$\Rightarrow P_A \Omega_A = \frac{\lambda^2}{4\pi S_B} P_B \Omega_B \tag{7.13}$$

Notice that

$$\frac{\lambda^2}{4\pi S_B} = \frac{1}{G_B} = 1 \tag{7.14}$$

Here, G_B is the gain of the bright body, which is unity because it was assumed that the body radiates isotropically. Thus,

$$P_A \Omega_A = P_B \Omega_B, \quad \text{if } \Omega_B \ll \Omega_A \tag{7.15}$$

Equation (7.15) leads directly to the relation between the brightness temperature of the observed object and the differential antenna temperature that is measured at the antenna terminals:

$$\Delta T_A = \frac{\Omega_B}{\Omega_A} T_B, \text{ K} \tag{7.16}$$

In general, the total antenna noise power will be made up by the weighted contributions of various sources whose temperature will vary with the angle of observation (θ,φ). The weighting factor is the power pattern of the antenna $F(\theta,\varphi)$.

$$T_A = \frac{1}{\Omega_A} \oiint_{4\pi} F(\theta,\varphi) \cdot T_B(\theta,\varphi) d\Omega \qquad (7.17)$$

Expression (7.17) is general, and the previously discussed special cases are easily derived from it. For example, assume that the brightness temperature surrounding the antenna is the same at all observation angles, $T_B(\theta,\varphi) = const = T_{B0}$. Then,

$$T_A = \frac{T_{B0}}{\Omega_A} \cdot \underbrace{\oiint_{4\pi} F(\theta,\varphi) d\Omega}_{\Omega_A} = T_{B0} \qquad (7.18)$$

The above situation has already been discussed in TP8 and TP9.

Assume now that $T_B(\theta,\varphi) = const = T_{B0}$ but only inside a solid angle Ω_B, which is much smaller than the antenna solid angle Ω_A. Outside Ω_B, $T_B(\theta,\varphi) = 0$. Since $\Omega_B \ll \Omega_A$, when the antenna is pointed at the noise source, its normalized power pattern within Ω_B is $F(\theta,\varphi) \simeq 1$. Then,

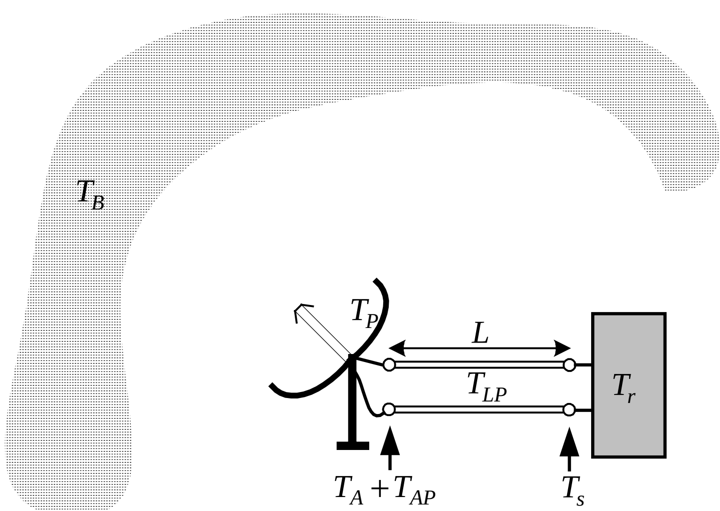
$$T_A = \frac{1}{\Omega_A} \oiint_{4\pi} F(\theta,\varphi) \cdot T_B(\theta,\varphi) d\Omega = \frac{1}{\Omega_A} \iint_{\Omega_B} 1 \cdot T_{B0} \cdot d\Omega = T_{B0} \frac{\Omega_B}{\Omega_A} \qquad (7.19)$$

This case has already been discussed in TP14, TP15 and TP16.

The antenna pattern strongly influences the antenna temperature. High-gain antennas (such as reflector systems), when pointed at elevation angles close to zenith, have negligibly low noise level. However, if an antenna has significant side and back lobes, which are pointed toward the ground or the horizon, its noise power is much higher. The worst case for an antenna is when its main beam points towards the ground or the horizon, as is often the case with radar and radio relay station antennas.

2. System temperature

An antenna is a part of a receiving system, which, in general, consists of a receiver, a transmission line and an antenna. All these system components have their contribution to the system noise. The system temperature (or the system noise level) is a critical factor in determining the sensitivity and the SNR of a receiving system.



If the antenna has some losses, then, at its terminals, the noise temperature consists of two terms: the antenna temperature due to the environment surrounding the antenna T_A, and the temperature due to the physical temperature of the antenna, T_{AP}. The last term is related to the physical temperature of the antenna T_P as:

$$T_{AP} = \left(\frac{1}{e_A} - 1 \right) T_P, \text{ K} \tag{7.20}$$

e_A is the *thermal efficiency* of the antenna $(0 \le e_A \le 1)$.

The transmission line itself is a source of noise if it has conduction losses. Its noise contribution at the antenna's terminals is:

$$T_L = \left(\frac{1}{e_L} - 1 \right) T_{LP}, \text{ K} \tag{7.21}$$

Here, $e_L = e^{-2\alpha L}$ is the *line thermal efficiency* $(0 \le e_L \le 1)$, T_{LP} is the physical temperature of the transmission line, α (Np/m) is the attenuation constant of the transmission line, and *L* is the length of the transmission line.

The total antenna noise power (which is proportional to the total antenna noise temperature) is transferred to the receiver's input through a transmission line, which is, in general, lossy with certain attenuation constant α (Np/m). So is the transmission-line noise power, as calculated with respect to the antenna terminals. Thus, at the receiver's input, the noise temperature due to the antenna and the transmission line is

$$T_{AL} = (T_A + T_{AP} + T_L) \cdot e^{-2\alpha L}, \text{ K} \tag{7.22}$$

$$\Rightarrow T_{AL} = (T_A + T_{AP}) \cdot e^{-2\alpha L} + T_{LP}(1 - e^{-2\alpha L}), \text{ K} \tag{7.23}$$

To the noise temperature calculated in (7.23) one has to add that of the receiver T_r. Thus, the system noise temperature is calculated as:

$$T_S = (T_A + T_{AP}) \cdot e^{-2\alpha L} + T_{LP}(1 - e^{-2\alpha L}) + T_r \text{, K} \tag{7.24}$$

The receiver's noise temperature is given by:

$$T_r = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \cdots \text{, K} \tag{7.25}$$

where:

T_1 is the noise temperature of the first amplifying stage;
G_1 is the gain of the first amplifying stage;
T_2 is the noise temperature of the second amplifying stage;
G_2 is the gain of the second amplifying stage.

3. System signal-to-noise ratio (SNR)

The system noise power is related to the system noise temperature as:

$$P_N = kT_s \vartriangle f \text{, W} \tag{7.26}$$

From Friis transmission equation:

$$P_r = (1 - |\Gamma_t|^2)(1 - |\Gamma_r|^2) e_t e_r |\hat{\rho}_t \cdot \hat{\rho}_r|^2 \left(\frac{\lambda}{4\pi R}\right)^2 D_t(\theta_t, \varphi_t) D_r(\theta_r, \varphi_r) \cdot P_t \tag{7.27}$$

one can calculate the signal power P_r. Thus, the SNR ratio becomes:

$$SNR = \frac{P_r}{P_N} = \frac{(1 - |\Gamma_t|^2)(1 - |\Gamma_r|^2) e_t e_r |\hat{\rho}_t \cdot \hat{\rho}_r|^2 \left(\frac{\lambda}{4\pi R}\right)^2 D_t D_r \cdot P_t}{kT_s \vartriangle f} \tag{7.28}$$

The above equation is fundamental for the design of telecommunication systems.