A Stochastic Multiple-Input-Multiple-Output Radio Channel Model for Evaluation of Space-Time Coding Algorithms

Klaus I. Pedersen[1], Jørgen Bach Andersen[2], Jean Philippe Kermoal[2], Preben Mogensen[1,2]

Nokia Networks[1]
Niels Jernesvej 10, DK-9220 Aalborg, Denmark
Email: Klaus.I.Pedersen@nokia.com

Center for PersonKommunikation[2]
Aalborg University, Fr. Bajersvej 7A, DK-9220 Aalborg, Denmark
Email: {jba,jpk,pm}@cpk.auc.dk

Abstract— **A simple framework for Monte-Carlo simulations of a multiple-input-multiple-output radio channel is proposed. The derived model includes the partial correlation between the paths in the channel, as well as fast fading and time dispersion. The only input parameters required for the model are the shape of the power delay spectrum and the spatial correlation functions at the transmit and receive end. Thus, the required parameters are available in the open literature for a large variety of environments. It is furthermore demonstrated that the Shannon capacity of the channel is highly dependent on the considered environment.**

I. Introduction

The remarkable Shannon capacity gains available from deploying multiple antennas at both the transmitter and receiver of a wireless system, has generated great interest in recent years [1]-[2]. Large capacity is obtained via the potential decorrelation in the multiple-input-multiple-output (MIMO) radio channel, which can be exploited to create many parallel subchannels. However, the potential capacity gain is highly dependent on the multipath richness in the radio channel, since a fully correlated MIMO channel only offers one subchannel, while a completely decorrelated channel offers multiple subchannels depending on the antenna configuration. Today, most studies have been conducted assuming either fully correlated/decorrelated channels, while a partially correlated channel should be expected in practice. The objective in this study is therefore to derive a realistic MIMO channel model, which is applicable for link level simulations for evaluation of space-time coding algorithms. It is believed that simulations are required in order to determine the performance of various space-time coding algorithms under more realistic propagation conditions, including channel estimation errors and other algorithmic imperfections.

During the last decade, there have been many studies focusing on single-input-multiple-output (SIMO) radio channel models for evaluation of adaptive antennas at the base station [3]. These models have mainly been derived from geometric scattering distributions [4]-[8] or extensive analysis of measurement data [9]-[11]. In this study, the goal is to take advantage of the numerous results obtained from studying SIMO channels, and try to extrapolate these findings into a simple wideband stochastic MIMO channel model. Measurement results which support the MIMO channel model derived in this paper are presented in [12].

II. Stochastic MIMO Channel

Let us consider the setup pictured in Fig. 1 with M antennas at the base station (BS) and N antennas at the mobile station (MS). The signals at the BS antenna array are denoted $\mathbf{y}(t) = [y_1(t), y_2(t), \ldots, y_M(t)]^T$, where $y_m(t)$ is the signal at the mth antenna port and $[\cdot]^T$ denotes transposition. Similarly, the signals at the MS are the components of the vector $\mathbf{s}(t) = [s_1(t), s_2(t), \ldots, s_N(t)]^T$.

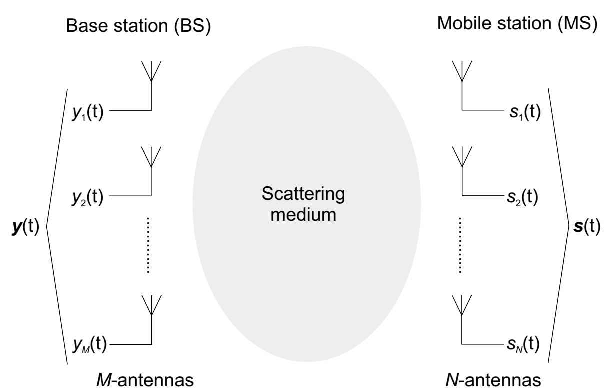

Fig. 1. Two antenna arrays in a scattering environment.

The wideband MIMO radio channel which describes the connection between the MS and the BS can be expressed as

$$\mathbf{H}(\tau) = \sum_{l=1}^{L} \mathbf{A}_l \delta(\tau - \tau_l), \qquad (1)$$

where $\mathbf{H}(\tau) \in \mathbb{C}^{M \times N}$ and

$$\mathbf{A}_l = \begin{bmatrix} \alpha_{11}^{(l)} & \alpha_{12}^{(l)} & \cdots & \alpha_{1N}^{(l)} \\ \alpha_{21}^{(l)} & \alpha_{22}^{(l)} & \cdots & \alpha_{2N}^{(l)} \\ \vdots & \vdots & & \vdots \\ \alpha_{M1}^{(l)} & \alpha_{M2}^{(l)} & \cdots & \alpha_{MN}^{(l)} \end{bmatrix}_{M \times N} \tag{2}$$

is a complex matrix which describes the linear transformation between the two considered antenna arrays at delay τ_l, where $\alpha_{nm}^{(l)}$ is the complex transmission coefficient from antenna n at the MS to antenna m at the BS. Notice that (1) is a simple tapped delay line model, where the channel coefficients at the L delays are represented by matrices. The relation between the vectors $\mathbf{y}(t)$ and $\mathbf{s}(t)$ can thus be expressed as

$$\mathbf{y}(t) = \int \mathbf{H}(\tau)\mathbf{s}(t-\tau)d\tau \tag{3}$$

or

$$\mathbf{s}(t) = \int \mathbf{H}^T(\tau)\mathbf{y}(t-\tau)d\tau, \tag{4}$$

depending on whether the transmission is from MS to BS, or vice versa.

To keep the channel model simple, it is assumed that $\alpha_{mn}^{(l)}$ is zero-mean complex Gaussian distributed, i.e. $|\alpha_{mn}^{(l)}|$ is Rayleigh distributed. It is furthermore assumed that the average power of the transmission coefficients is identical for a given delay, so

$$P_l = \mathrm{E}\left\{|\alpha_{mn}^{(l)}|^2\right\} \tag{5}$$

for all

$$n \in [1, 2, \ldots, N] \;\; \wedge \;\; m \in [1, 2, \ldots, M], \tag{6}$$

and uncorrelated from one delay to another, so

$$\left\langle |\alpha_{mn}^{(l_1)}|^2, |\alpha_{mn}^{(l_2)}|^2 \right\rangle = 0 \quad \text{for} \;\; l_1 \neq l_2, \tag{7}$$

where $\langle a, b \rangle$ computes the correlation coefficient between a and b. This implies that the average power delay spectrum (PDS) that the model reproduce equals $P(\tau) = \sum P_l \delta(\tau - \tau_l)$. Thus, by proper selection of the parameter set $\{\tau_l, P_l\}$ an exponential decaying PDS with a specified delay spread can be realized [17]. Alternatively, the shape of $P(\tau)$ can be chosen according to some of the specified ITU profiles such as Vehicular A, Indoor A, Pedestrian A, etc. [18].

The potential gain from applying space-time coding is strongly dependent on the correlation coefficient between the components of $\mathbf{H}(\tau)$ and thus of $\mathbf{A}_l$. The spatial correlation function observed at the BS has been studied extensively in the literature for scenarios where the MS is surrounded by scatterers, while there are no local scatterers in the vicinity of the BS antenna array, i.e. typical urban environment. This basically means that the power azimuth spectrum (PAS) observed at the BS is confined to a relatively narrow beamwidth. Given these conditions, expressions for the spatial correlation function have been derived assuming that the PAS follows a cosine raised to an even integer [13], a Gaussian function [14], a uniform function [15], and a Laplacian function [16]. Experimental results have been reported in [19]-[20], among others. Consequently, the correlation coefficient between antenna m_1 and m_2 at the BS,

$$\rho_{m_1 m_2}^{\text{BS}} = \left\langle |\alpha_{m_1 n}^{(l)}|^2, |\alpha_{m_2 n}^{(l)}|^2 \right\rangle \tag{8}$$

is easily obtained from the literature assuming that the BS antenna array is elevated above the local scatterers. Notice that (8) implicitly assumes that the spatial correlation function at the BS is independent of n. This is a reasonable assumption provided that all antennas at the MS are closely co-located and have the same radiation pattern, so they illuminate the same surrounding scatterers and therefore also generate the same PAS at the BS, i.e. the same spatial correlation function.

The spatial power correlation function observed at the MS has also been extensively studied in the literature [25]-[27], among others. Assuming that the MS is surrounded by local scatterers, antennas separated more than half a wavelength can be assumed to be practically uncorrelated [25], with

$$\rho_{n_1 n_2}^{\text{MS}} = \left\langle |\alpha_{m n_1}^{(l)}|^2, |\alpha_{m n_2}^{(l)}|^2 \right\rangle \tag{9}$$

approximately zero for $n_1 \neq n_2$. However, experimental results reported in [9] show that in some situations antennas separated half a wavelength can be highly correlated for a MS located in an indoor environment. Under such conditions, an approximate expression of the spatial correlation function averaged over all possible azimuth orientations of the MS array is derived in [28]. This expression is a function of the azimuth dispersion $\Lambda \in [0; 1]$, where $\Lambda = 0$ corresponds to a scenario where the power is coming from one distinct direction only, while $\Lambda = 1$ when the PAS is uniformly distributed over the azimuthal range $[0°; 360°[$ [29]. As the MS typically is non-stationary, the results presented in [28] are considered to be very useful since they are averaged over all orientations of the MS array.

Given (8) and (9), let us define the symmetrical correlation matrices

$$\mathbf{R}_{\text{BS}} = \begin{bmatrix} \rho_{11}^{\text{BS}} & \rho_{12}^{\text{BS}} & \cdots & \rho_{1M}^{\text{BS}} \\ \rho_{21}^{\text{BS}} & \rho_{22}^{\text{BS}} & \cdots & \rho_{2M}^{\text{BS}} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{M1}^{\text{BS}} & \rho_{M2}^{\text{BS}} & \cdots & \rho_{MM}^{\text{BS}} \end{bmatrix}_{M \times M} \tag{10}$$

$$\mathbf{R}_{\text{MS}} = \begin{bmatrix} \rho_{11}^{\text{MS}} & \rho_{12}^{\text{MS}} & \cdots & \rho_{1N}^{\text{MS}} \\ \rho_{21}^{\text{MS}} & \rho_{22}^{\text{MS}} & \cdots & \rho_{2N}^{\text{MS}} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{N1}^{\text{MS}} & \rho_{N2}^{\text{MS}} & \cdots & \rho_{NN}^{\text{MS}} \end{bmatrix}_{N \times N} \tag{11}$$

for later use.

The spatial correlation function at the BS and the MS does not provide sufficient information to generate the matrices $\mathbf{A}_l$. The correlation of two transmission coefficients connecting two different sets of antennas also need to be determined, i.e.

$$\rho^{n_1 m_1}_{n_2 m_2} = \left\langle |\alpha^{(l)}_{m_1 n_1}|^2, |\alpha^{(l)}_{m_2 n_2}|^2 \right\rangle \tag{12}$$

for $n_1 \neq n_2$ and $m_1 \neq m_2$. However, it can be shown theoretically that

$$\rho^{n_1 m_1}_{n_2 m_2} = \rho^{\mathrm{MS}}_{n_1 n_2} \rho^{\mathrm{BS}}_{m_1 m_2}, \tag{13}$$

provided that (8) and (9) are independent of n and m, respectively.

III. Simulation of the MIMO Channel

A. Generation of correlated fading

Given the proposed model, the MIMO channel is easily simulated on a computer by generating LMN uncorrelated complex Gaussian processes followed by a filtering procedure in order to obtain the desired correlation between the transmission coefficients. Following the approach in [22], the correlated transmission coefficients can be obtained according to

$$\tilde{\mathbf{A}}_l = \sqrt{P_l}\mathbf{C}\mathbf{a}_l \tag{14}$$

where

$$\tilde{\mathbf{A}}_l = \begin{bmatrix} \alpha^{(l)}_{11} \\ \alpha^{(l)}_{21} \\ \vdots \\ \alpha^{(l)}_{M1} \\ \alpha^{(l)}_{12} \\ \alpha^{(l)}_{22} \\ \vdots \\ \alpha^{(l)}_{M2} \\ \alpha^{(l)}_{13} \\ \vdots \\ \alpha^{(l)}_{MN} \end{bmatrix}_{MN\times 1} \qquad \mathbf{a}_l = \begin{bmatrix} a^{(l)}_{1} \\ a^{(l)}_{2} \\ a^{(l)}_{3} \\ \vdots \\ a^{(l)}_{MN} \end{bmatrix}_{MN\times 1} \tag{15}$$

where it is assumed that $a^{(l)}_x$ is zero-mean complex Gaussian distributed with $\mathrm{E}\{|a^{(l)}_x|^2\} = 1$ and $\langle |a^{(l_1)}_{x_1}|^2, |a^{(l_2)}_{x_2}|^2 \rangle = 0$ for $x_1 \neq x_2$ or $l_1 \neq l_2$. The symmetrical mapping matrix $\mathbf{C} \in \mathbb{R}^{MN\times MN}$, results in a correlation matrix $\mathbf{\Gamma} = \mathbf{C}\mathbf{C}^T$ where the (x, y)-th element of $\mathbf{\Gamma}$ is the root power correlation coefficient between the xth and yth element of $\tilde{\mathbf{A}}_l$. As a first approximation, it is proposed to simulate $a^{(l)}_x$ according to Jakes' model [26] to ensure that temporal correlation is included in the model as well. Assuming that the components of $\mathbf{\Gamma}$ are known, the symmetrical mapping matrix $\mathbf{C}$ can be obtained by using a standard matrix square root decomposition method as described in [21], provided that $\mathbf{\Gamma}$ is non-singular.

B. Directional properties

Notice that the proposed model only reproduces the correlation metrics and fast fading characteristics of the radio channel, while the phase derivative across the antenna arrays is not necessarily reflected correctly in the model. The current model gives rise to a mean phase variation of $0°$ across the antenna array, provided that the elements are highly correlated. The implication is a mean direction-of-arrival (DoA) of the impinging field corresponding to broadside, since the phase difference between two antenna elements is proportional to $\sin(\phi)$, where ϕ is the DoA. However, the model is easily modified to comply with scenarios like the one pictured in Fig. 2, where the mean DoA at the BS $\overline{\phi}_{\mathrm{BS}} \neq 0°$. This mechanism is included in the model by modifying (3) to

$$\mathbf{y}(t) = \mathbf{W}(\overline{\phi}_{\mathrm{BS}}) \int \mathbf{H}(\tau)\mathbf{s}(t-\tau)d\tau \tag{16}$$

where the steering diagonal matrix is expressed as

$$\mathbf{W}(\phi) = \begin{bmatrix} w_1(\phi) & 0 & \cdots & 0 \\ 0 & w_2(\phi) & & \vdots \\ \vdots & & \ddots & \\ 0 & \cdots & & w_M(\phi) \end{bmatrix}_{M\times M}, \tag{17}$$

where $w_m(\phi)$ describes the average phase shift relative to antenna number one assuming that the mean azimuth DoA of the impinging field equals ϕ. For a uniform linear antenna array with element spacing d, $w_m(\phi) = f_m(\phi)\exp[-j(m-1)d\lambda^{-1}2\pi\sin(\phi)]$ where $f_m(\phi)$ is the complex radiation pattern of antenna m, λ is the wavelength, and j is the imaginary unit.

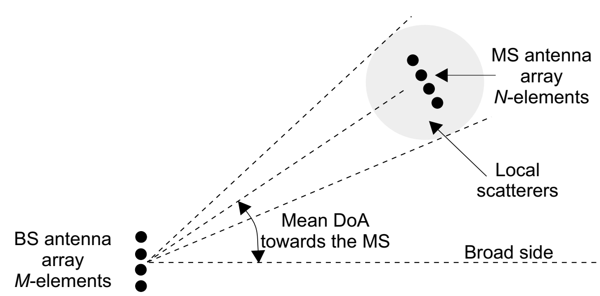

Fig. 2. Sketch of a scenario where all scatterers are located near the MS so the impinging filed at the BS is confined to a narrow azimuth region with a well defined mean direction-of-arrival.

In situations where the antenna signals at the array are assumed statistically independent (uncorrelated), the model reproduces random phase variations from one antenna to another. In such environments, it does not make sense to define a mean DoA, hence (3) is applicable without the modification proposed in (16).

C. Simulation flow

Simulation of the proposed MIMO channel on a computer basically consists of two steps as illustrated in Figs. 3a and

3b. In the initialization phase, a given environment is first selected, where the power delay spectrum and the spatial correlation functions at the MS and BS are known a priori. Given the antenna configuration at the MS and BS, the correlation matrices $\mathbf{R}_{\text{MS}}$ and $\mathbf{R}_{\text{BS}}$ are determined, and subsequently the mapping matrix $\mathbf{C}$ can be computed. Notice that $\mathbf{C}$ only needs to be computed once per simulation.

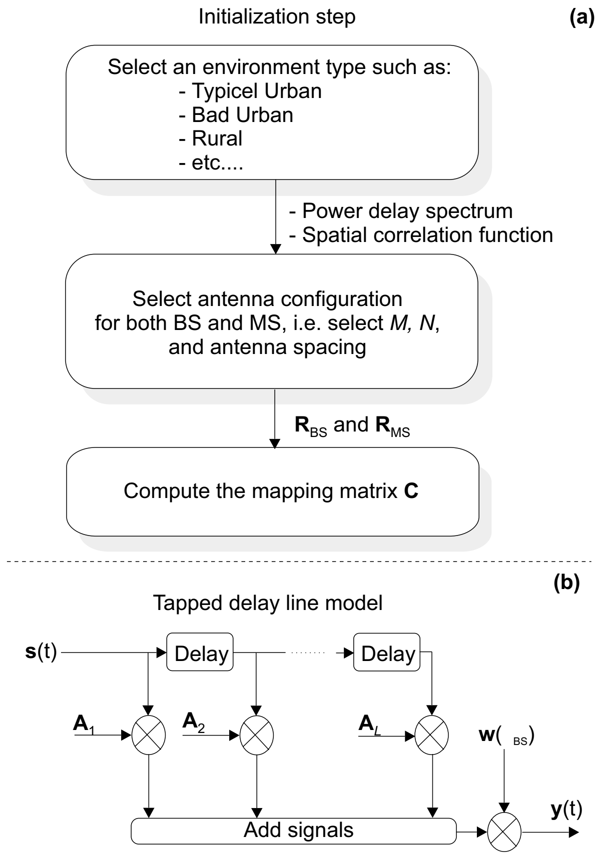

Fig. 3. Implementation of the MIMO channel; (a) initialization steps (b) Tapped delay line model.

After the initialization phase, the MIMO channel is simulated as a tapped delay line as shown in Fig. 3b. Additional details are discussed more thoroughly in [31], where a COSSAP implementation is presented.

IV. Some Capacity Results

To illustrate the behavior of the proposed MIMO channel, let us consider a scenario with $N = 2$ antennas at the MS and $M = 4$ antennas at the BS. Uniform linear arrays are assumed at both ends with half a wavelength element spacing. A typical urban environment is considered with the MS situated in a street canyon and the BS antenna array mounted slightly above rooftop level of the surrounding buildings. No line-of-sight exists between the MS and BS. The PAS observed at the BS typically follows a Laplacian function for such a typical urban environment [23]-[24], with a mean azimuth spread (AS) on the order of 10° [16]. Consequently, an analytical expression for the spatial correlation function at the BS is available in [16], which yields

$$\mathbf{R}_{\text{BS}} = \begin{bmatrix} 1.00 & 0.91 & 0.73 & 0.46 \\ 0.91 & 1.00 & 0.91 & 0.73 \\ 0.73 & 0.91 & 1.00 & 0.91 \\ 0.46 & 0.73 & 0.91 & 1.00 \end{bmatrix}. \qquad (18)$$

This implies that two neighboring elements are highly correlated (0.91), while the two outer elements in the BS array exhibit low correlation (0.43). The MS is assumed to be surrounded by numerous local scatterers which results in a fairly low correlation between the two antennas [25]. Hence, as an example we can assume

$$\mathbf{R}_{\text{MS}} = \begin{bmatrix} 1.0 & 0.3 \\ 0.3 & 1.0 \end{bmatrix}. \qquad (19)$$

Given this channel configuration, let us determine the potential capacity for a narrowband scenario ($L = 1$). The MIMO channel $\mathbf{H}(\tau)$ offers K separate data subchannels, where K is the rank of $\mathbf{R} = \mathbf{H}\mathbf{H}^H$ and bounded by $K \leq \min\{N, M, Q\}$ assuming that there are Q discrete propagation paths present in the channel [2]. Here $[\cdot]^H$ denotes Hermitian transposition and the function $\min\{\cdot\}$ returns the smallest argument. The gain in the kth subchannel is given by the kth eigenvalue λ_k of $\mathbf{R}$. Assuming that equal power is allocated to each subchannel[1], the total capacity can be expressed by applying Shannon's capacity formula,

$$\begin{aligned} C &= \sum_{k=1}^{K} \log_2 \left(1 + \frac{\lambda_k}{K} \frac{P}{\sigma^2} \right) & (20) \\ &= \log_2 \left| \mathbf{I} + \mathbf{R} \frac{P}{K\sigma^2} \right|, & (21) \end{aligned}$$

where P is the total power, σ^2 is the noise power, $\mathbf{I}$ is the Identity matrix, and $|\cdot|$ is the Determinant. Based on (21), the cumulative distribution function (cdf) of C is obtained via Monte-Carlo simulations for three different cases: (i) All transmission coefficients are completely correlated, (ii) fully uncorrelated, and (iii) partially correlation as specified in (18)-(19). Notice that case (i) might correspond to a situation where there are few scatterers and a LOS between the MS and BS, say a macro cellular rural environment. Case (ii) is equivalent to an indoor pico cell configuration where both antenna arrays are surrounded by a large number of local scatterers. The cdf's of C are plotted in Fig. 4 for the three different cases assuming a signal-to-noise ratio (SNR) of $P/\sigma^2 = 20.0$ dB.

It is observed that the potential capacity is very sensitive to the mutual correlation of the transmission coefficients and thus the type of environment. The capacity almost doubles when comparing the correlated and uncorrelated case at the 10% outage level. The capacity of the partially correlated channel (typical urban) is somewhere in between the two extreme cases.

[1] If optimum power allocation is applied instead, then it can be obtained by using the *Water filling* theorem as discussed in [30].

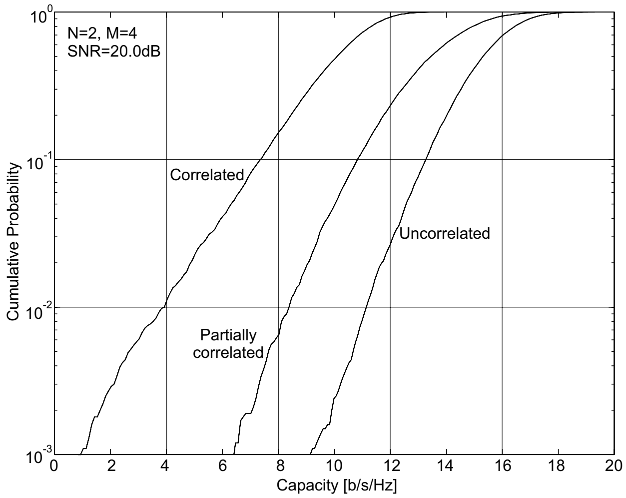

Fig. 4. Cumulative distribution function of the capacity for three different propagation scenarios.

V. Concluding Remarks

The proposed MIMO channel model provides a simple framework for simulation of such channels, and allows existing wideband tapped delay line single-input-single-output channel models to be easily extended to include MIMO. The model is designed so the required parameters are accessible in the open literature for various types of environments.

VI. Acknowledgement

The presented results are obtained as part of the European Union IST research project METRA (Multi Element Transmit Receive Antennas). This project includes partners from Universitat Politecnica de Catalunya, Spain, Vodafone Limited, UK, Nokia Networks and Nokia Mobile Phones, Finland, and Center for PersonKommunikation at Aalborg University, Denmark.

References

[1] G.J. Foschini, “Layered Space-Time Architecture for Wireless Communication in Fading Environment When Using Multi-Element Antennas”, *Bell Labs Technical Journal*, pp. 41-59, Autumn 1996
[2] G.G. Raleigh, J.M. Cioffi, “Spatio-Temporal Coding for Wireless Communication”, *IEEE Trans. on Communications*, Vol. 46, No. 3, pp. 357-366, March 1998.
[3] R.B. Ertel, P. Cardieri, K.W. Sowerby, T.S. Rappaport, J.H. Reed, “Overview of Spatial Channel Models for Antenna Array Communication Systems”, *IEEE Personal Communications*, pp. 10-21, February 1998.
[4] J. Liberti, T. Rappaport, “A Geometrically Based Model for Line-Of-Sight Multipath Radio Channels”, *IEEE Proc. Vehicular Technology Conference (VTC96)*, pp. 844-848, May 1996.
[5] P. Petrus, J.H. Reed, T.S. Rappaport, “Geometrically Based Statistical Model for Macrocellular Mobile Environments”, *IEEE Proc. GLOBECOM’96*, London, UK, pp. 1197-1201, November 1996.
[6] M. Lu, T. Lo, J. Litva, “A Physical Spatio-Temporal Model of Multipath Propagation Channels”, *IEEE Proc. Vehicular Technology Conference (VTC97)*, pp. 810-814, May 1997.
[7] J. Fuhl, A.F. Molisch, E. Bonek “A Unified Channel Model for Mobile Radio Systems with Smart Antennas”, *IEE Proc. Radar, Sonar, and Navigation*, Vol. 145, No. 1, pp. 32-42, Februar 1998.
[8] O. Nørklit, J. Bach Andersen, “Diffuse Channel Model and Experimental Results for Antenna Arrays in Mobile Environments”, *IEEE Trans. on Antennas and Propagation*, Vol. 46, No. 6, pp. 834-840, June 1998.
[9] P. Eggers, “Angular Dispersive Mobile Radio Environments Sensed by Highly Directive Base Station Antennas”, *IEEE Proc. Personal, Indoor and Mobile Radio Communications*, pp. 522-526, September 1995.
[10] K.I. Pedersen, P.E. Mogensen, B. Fleury, “A Stochastic Model of the Temporal and Azimuthal Dispersion seen at the Base Station in Outdoor Propagation Environments”, *IEEE Trans. on Vehicular Technology*, Vol. 49, No. 2, pp. 437-447, March 2000.
[11] K.I. Pedersen, P.E. Mogensen, “Simulation of Dual-Polarized Propagation Environments for Adaptive Antennas”, *IEEE Proc. Vehicular Technology Conference*, pp. 62-66, September 1999.
[12] J.P. Kermoal, K.I. Pedersen, P.E. Mogensen, ”Experimental Investigation of Correlation Properties of MIMO Radio Channels for Indoor Picocell Scenario”, *VTC-2000 Boston, (In this proceeding)*.
[13] W. Lee, “Effects on Correlation Between Two Mobile Radio Base-Station Antennas”, *IEEE Trans. on Communications*, Vol. 21, No. 11, pp. 1214-1224, November 1973.
[14] F. Adachi, M. Feeny, A. Williamson, J. Parsons, “Crosscorrelation between the envelopes of 900MHz signals received at a mobile radio base station site”, *IEE Proc. Pt. F.*, Vol. 133, No. 6, pp. 506-512, October 1986.
[15] J. Salz, J. Winters, “Effect of Fading Correlation on Adaptive Arrays in Digital Mobile Radio”, *IEEE Trans. on Vehicular Technology*, Vol. 43, No. 4, pp. 1049-1057, November 1994.
[16] K.I. Pedersen, P.E. Mogensen, B.H. Fleury, “Spatial Channel Characteristics in Outdoor Environments and their Impact on BS Antenna System Performance”, *IEEE Proc. Vehicular Technology Conference (VTC’98)*, Ottawa, Canada, pp. 719-724, May 1998.
[17] Cost207, “Information Technologies and Sciences - Digital Land Mobile Radio Communications”, Commission of the European Communities, September 1988.
[18] Universal Mobile Telecommunications System (UMTS), “Selection procedure for the choice of radio transmission technologies of UMTS”, *UMTS 30.03 version 3.2.0 ETSI*, April 1998.
[19] P.C.F. Eggers, J. Toftgaard, A.M. Oprea, “Antenna Systems for Base Station Diversity in Urban Small and Micro Cells”, *IEEE Journal on Selected Areas in Communications*, Vol. 11, No. 7, pp. 1046-1057, September 1993.
[20] T.B. Sørensen, A.Ø. Nielsen, P.E. Mogensen, “ Performance of Two-Branch Polarization Antenna Diversity in an Operational GSM Network”, *IEEE Proc. VTC’98*, pp. 741-746, May 1998.
[21] G.H. Golub, C.F. Van Loan, “Matrix Computations”, *Third Edition, The Johns Hopskins University Press*, 1996.
[22] T. Klingenbrunn, P.E. Mogensen, “Modelling Frequency Correlation of Fast Fading in Frequency Hopping GSM Link Simulations”, *IEEE Proc. Vehicular Technology Conference*, pp. 2398-2402, September 1999.
[23] K.I. Pedersen, P.E. Mogensen, B.H. Fleury, “Power Azimuth Spectrum in Outdoor Environments”, *IEE Electronics Letters*, Vol. 33, No. 18, pp. 1583-1584, 28th August 1997.
[24] U. Martin, “A Directional Radio Channel Model for Densely Built-Up Urban Areas”, *Proc. 2nd EPMCC*, Bonn, Germany, pp. 237-244, October 1997.
[25] R.H. Clark, “A Statistical Theory of Mobile Radio Reception”, *Bell Labs System Technical Journal*, Vol. 47, pp. 957-1000, July-August 1968.
[26] W.C. Jakes, “Microwave Mobile Communications”, *IEEE Press*, 1974.
[27] T. Aulin, “A Modified Model for the Fading Signal at a Mobile Radio Channel”, *IEEE Trans. on Vehicular Technology*, Vol. 28, No. 3, pp. 182-203, August 1979.
[28] G. Durgin, T.S. Rappaport, “Effects of Multipath Angular Spread on the Spatial Cross-Correlation of Received Voltage Envelopes”, *IEEE Proc. Vehicular Technology Conference*, pp. 996-1000, May 1999.
[29] G. Durgin, T.S. Rappaport, “Basic relationship between multipath angular spread and narrowband fading in wireless channels”, *IEE Electronics Letters*, Vol. 34, No. 25, pp. 2431-2432, December 1998.
[30] J. Bach Andersen, “Array Gain and Capacity for Known Random Channels with Multiple Element Arrays at Both Ends”, *Journal on Selected Areas in Communications*, Accepted: To appear 2000.
[31] L. Schumacher, K.I. Pedersen, J.P. Kermoal, P.E. Mogensen, “A Link-Level Simulator Implementing a Stochastic MIMO Radio Channel Model for Evaluation of Combined Transmit/Receive Diversity Concepts within the METRA Project”, *To appear: IST Mobile Communications Summit*, Ireland, October 2000.