Microdiversity Reception of Spread-Spectrum Signals on Nakagami Fading Channels

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Abstract—An analytical framework to evaluate the performance of different predetection diversity techniques in various mobile radio environments is developed. The average bit-error rate analysis applies to phase coded spread-spectrum systems, over Nakagami multipath fading channels. A simple and practical selection combining rule is considered. Our numerical results reveal that this new low-complexity receiver structure exhibits comparable performance to that of an optimum linear diversity combiner when the channel does not experience severe fading and for small diversity orders, conditioned on the situation that all the diversity branches have identical mean signal strengths. In this study, we also investigate the effect of variations in the mean signal and noise power levels on each of the independent diversity branches. This is an important consideration because in practice equal mean signal strengths rarely occur, which results in loss of diversity gain. We found that the signal-plus-noise-and-interference selection model outperforms the traditional signal-to-interference-plus-noise ratio selection scheme if the discrepancy between the mean signal strengths are small, owing to the statistical nature of the multiple-access interference.

Index Terms—Diversity methods, mobile communication, Nakagami multipath fading channels, spread-spectrum communication.

I. INTRODUCTION

DIVERSITY reception has long been recognized as an effective technique for combating the detrimental effects of channel fading. The underlying premise is that if several uncorrelated replicas of a signal are received over multiple diversity branches with comparable strengths, then it is probable that these signals will experience simultaneous deep fades. Diversity methods can be employed either at the base station (macroscopic diversity) or at the mobile (microscopic diversity or microdiversity), although the antenna separation required differs for each case. In practice, microdiversity reception techniques are employed to combat the fast fading variations in the received signal strength caused by multipath fading, whereas macrodiversity is used to mitigate the slower fading variations caused by shadowing.

To capitalize on the improvements in signal statistics due to diversity, several combining techniques have been proposed, and they can be categorized into two main groups, namely switched combining and gain combining. Among these approaches, selecting the best diversity reception is the simplest and perhaps the most frequently used form of diversity combining [1]. It is also worth mentioning here that the improvements achieved through diversity reception are usually at the expense of increased system complexity. For instance, in a signal-to-interference-plus-noise ratio (SINR) based diversity scheme, some type of SINR measuring device is required or a pilot tone calibration scheme may be employed [2]. Therefore, techniques of improving the system performance without incurring a substantial penalty in terms of implementation complexity or cost are of practical interest.

The conventional selection diversity system measures the SINR at each branch (i.e., antenna) and chooses the branch with the highest SINR value for data recovery. Thus, if $M$th-order diversity is employed and the mean noise power is assumed to be identical for all branches, then the decision criteria reduces to $\max \{\alpha_i\}$, $i = 1, \ldots, M$, where $\alpha_i$ is the channel gain from the $i$th branch [3]. However in practice, measurements of SINR may be difficult or expensive [2], especially for high data rate transmissions. To be most effective, the system should be able to make its selection in a period of time equal to or less than the interval of the shortest signal that will be transmitted. Consequently, the branch with the largest amplitude of the received composite signal is chosen.

In a related work [3], Chyi et al. investigated the performance of two receiver structures for $M$-ary orthogonal signaling over frequency nonselective Rayleigh fading channels. They have shown that the conventional selection scheme which selects the branch with the highest SINR has inferior performance compared to that of a signal-plus-noise-and-interference ($S + I$) selection system. The analysis was extended in [4] by considering four receiver structures for binary noncoherent frequency-shift keying signaling and two receiver structures for binary differential phase-shift keying signaling employing selection diversity. Furthermore, an explanation for the rather interesting observation cited in [3] was furnished, with the reason being that the traditional selection diversity model does not take into account the stochastic nature of the noise. In a sequel study [5], we evaluated the performance of these two different selection diversity schemes for slotted direct-sequence/code-division multiple-access (DS/CDMA) packet radio communications over multipath Rayleigh fading channels. Our results confirm the observations presented in [3] and [4], for coherent signal reception. It should be emphasized, however, that the above
conclusions were drawn based on the assumption that all the statistically independent diversity branches have identical mean signal strengths. By contrast, in this paper we move away from this restriction and study the efficacy of selection diversity in more realistic situations. This is an important consideration because in practice equal mean signal strengths rarely occur, which results in loss of diversity gain.

Additionally, we extend the previous studies [4], [5] in several ways: 1) by studying the efficacy of selection diversity schemes in Nakagami multipath fading channels (i.e., investigate the effects of fade distributions on the attainable diversity improvement); 2) by deriving computationally efficient formulas for evaluating average probability of bit error in various mobile radio environments; and 3) by validating the observations cited in [4] for coherent binary phase-shift keying (BPSK) signaling. Analytical expressions are derived to evaluate the average bit-error probability of a phase-coded spread-spectrum (SS) receiver with antenna (spatial) diversity reception. In addition to the SINR and S + I selection methods, performance of an optimum linear diversity combiner is also evaluated for comparison. The analysis presented here is also valid for frequency and time-diversity schemes.

An outline of the paper is as follows. The system fading and multipath model is briefly described in Section II. Section III details the error performance analysis for a correlation receiver that combines the signals from multiple antennas using various predetection diversity techniques at the bit level. Diversity gain provided by two different selection combining rules and the maximum-ratio combining approach are investigated. Numerical results are illustrated in Section IV. Finally in Section V, the main points are summarized and conclusions restated.

II. SYSTEM MODEL

The system model to be considered consists of $K$ active users transmitting binary data simultaneously to a central
station (the first user being the reference user whose performance is to be evaluated), as depicted in Fig. 1(a), for a nondiversity receiver \((M = 1)\). We characterize the link between a transmitter and its receiver as a Nakagami multipath fading channel. Nakagami's \(m\)-distribution \([6]\) is a versatile statistical model (a generalized statistical distribution), which can accurately fit experimental data for many physical propagation channels. For instance, Suzuki \([7]\) and Braun and Dersch \([8]\) have shown that Nakagami distributions fit some urban radio multipath channels data better than Rayleigh, Rice, or log-normal distributions.

The multipath fading channel is modeled as frequency selective in that the chip rate is higher than the channel coherence bandwidth. In this case, the discrete multipath components in the received signal are resolvable with a resolution in time delay of \(T_c\), the chip duration. The multipath model used here is based on a random delay, discrete resolvable path model with unequal path strengths \([9]\). Therefore, the relative delays between the multipath components are not used. The random delay model seems better suited than the tapped delay line model \([10]\) for situations where there are only a few distinct paths available.

After the correlation operation that collapses the wide-band coded signal into a narrow-band modulated signal and the demodulation process, a signal sample at the receiver low-pass filter output can be expressed as

\[
\zeta_{11} = \beta_{11} \sqrt{\frac{P}{2}} T_b b_{1,0} + \sqrt{\frac{P}{2}} T_b (I_{\text{mu}} + I_{\text{mp}}) + \eta
\]

where \(\beta_{k1}\) denotes the \(k\)th path gain of the \(k\)th user, and \(T_b\) is the bit duration. \(P\) and \(b_{1,0}\) correspond to the average transmitted power and the polarity of the data bit to be detected, respectively. \(\eta = \int_{\Omega \Gamma} \tau(t) a_1(t) \cos(\omega_c t) dt\), where \(a_1(t)\) is the spreading waveform of user one, and \(\omega_c\) is the common angular carrier frequency.

\(I_{\text{mu}}\) in (1) corresponds to the multiuser interference

\[
I_{\text{mu}} = \sum_{k=2}^{K} \beta_{k1} \left[ b_{k-1} R_{k,k} (\tau_{k1}) + b_{k,0} \tilde{R}_{k,k} (\tau_{k1}) \right] \cos(\varphi_{k1})
\]

and \(I_{\text{mp}}\) is due to multipath interference

\[
I_{\text{mp}} = \sum_{k=1}^{K} \sum_{l=2}^{L} \beta_{kl} \left[ b_{k-1} R_{k,l} (\tau_{k1}) + b_{k,0} \tilde{R}_{k,l} (\tau_{k1}) \right] \cos(\varphi_{kl})
\]

Here, \(b_{k-1}\) and \(b_{k,0}\) denote the previous and current data bit of the \(k\)th user, respectively; and \(\varphi_{kl} = \theta_k + \varphi_{kl} - \omega_c T_{kl}\) where \(\theta_k\) is the phase angle introduced by the \(k\)th PSK modulator, and \(\varphi_{kl}\) corresponds to the phase delay. The continuous-time partial cross-correlation functions are defined as in \([12]\):

\[
R_{k,l}(\tau, \gamma) = \int_{-\infty}^{\infty} a_k(t) a_l(t - \tau) dt \quad \text{and} \quad \tilde{R}_{k,l}(\tau, \gamma) = \int_{-\infty}^{\infty} a_k(t) a_l(t - \tau) dt.
\]

To arrive at (1), we have assumed that average power control is employed, which means that on average, the same power is received from each active user. The first term in (1) represents the desired signal to be detected, while the final term is a Gaussian random variable with zero mean and variance (power) \(N_0 T_b/4\), due to additive white Gaussian noise (AWGN).

### III. Error Probability Analysis

In this section, analytical expressions are derived to evaluate the bit-error probabilities of a correlation receiver with antenna diversity reception, by modeling the multiple-access interference (MAI) as a Gaussian process. The standard Gaussian approximation method for MAI is very attractive due to its simplicity. It also yields reasonably accurate results, especially for high bit-error rates (BER’s), small SINR, and processing gain values, and if the SINR is conditioned on the fading \([13]\).

A prerequisite to the application of the Gaussian approximation is knowledge of the variance of the MAI. Variance of the cross-correlation function for Gold codes has been derived in \([12]\) and is given by

\[
\vartheta_{\mu} = \frac{2}{3N}
\]

where \(\alpha_y \in \{\pm 1\}, y = \{-1, 0\}\) are independently, identically distributed (i.i.d.) binary random variables. \(N\) denotes the system processing gain, and therefore each bit is encoded with \(N\) chips.

Following the central limit theorem, the total interference is assumed to be Gaussian \(^1\) distributed with zero mean and variance equal to the sum of variances of all the terms that contribute to MAI in (1). Hence, the signal-to-noise-plus-interference ratio is given by

\[
\gamma = \frac{\beta_{11}^2}{\frac{1}{3N} \left( K - 1 \right) \vartheta_{\mu} \beta_{11}^2 + \sum_{k=2}^{K} \sum_{l=2}^{L} \vartheta_{\mu} \beta_{kl}^2} + \frac{N_0}{2E_b} = 2\gamma_b
\]

where \(\gamma_b\) denotes the received signal-to-noise ratio (SNR) (usually referred to as SNR per bit \([14]\)). The sample (decision statistic) of the received composite signal described in (1) can

\(^1\)Computation of the self-multipath interference (self-noise) term inherent in (3) (i.e., for \(k = 1\)) is rather complicated because it is dependent on the adjacent bits as well as on the assigned signature code and partial autocorrelation properties. However, for large \(K\), the variance of the MAI will be dominated by the multiuser interference term because the magnitude of the self-noise will be very small relative to multiuser interference power. Therefore, an approximation for self-interference similar to that of multiuser interference seems quite reasonable. A detailed explanation can be found in \([10]\).
be further simplified and rewritten in a more compact form [15]

$$
\zeta_j \equiv \alpha_j b_0 + n_j
$$

where $\alpha_j = \sqrt{P_j T_j b_j}$ is the fading sample, $n_j$ is the MAI (multipath and multiuser interference plus AWGN noise) sample with variance $\sigma_n^2$, and $b_0$ is the polarity of the data bit being detected. Also, $\alpha_j$ and $n_j$ are assumed to be i.i.d. Nakagami and Gaussian random variables, respectively. Subscript $j$ corresponds to the $j$th copy of a signal available at the receiver for detection, $j \in \{1,2,\cdots,M\}$, where $M$ denotes the diversity order.

Since $\alpha_j$ is a Nakagami random variable, it has probability density function (pdf)

$$
f_\alpha(R) = \frac{2m^m R^{2m-1}}{\Gamma(m) \Omega^m} \exp \left( \frac{-mR^2}{\Omega} \right) u(R)
$$

where $u(\cdot)$ denotes the unit-step function and $\Gamma(\cdot)$ denotes the Gamma function. The parameter $m$ in (7) is assumed to be common for all diversity branches and is defined as the ratio of moments, called the fading figure [14], $m = \frac{\Omega^2}{E[R^2] - \Omega^2}$, where $\Omega = E[R^2]$, and $E(\cdot)$ represents the mean function.

**A. SINR Selection Diversity**

The conventional selection diversity model specifies that of $M$ diversity branches, the one providing the largest SINR be selected for data recovery. Improvement in signal reception is achieved by the uncorrelated nature of the signals from the multiple antennas, i.e., the likelihood that all of the $M$ signals fade simultaneously is much lower than the fading probability of any signal from a single branch. If we assume that $b_0 = +1$ was transmitted, then the conditional error probability is given by

$$
\text{Prob} [\zeta_j < 0 | b_0 = +1] = \sum_{j=1}^{M} \text{Prob} \left\{ \alpha_j + n_j < 0, \frac{\alpha_j^2}{\sigma_j^2} > \frac{\Omega^2}{\sigma_j^2} \right\},
$$

and

$$
= \prod_{i=1, i \neq j}^{M} \frac{1}{1 - \exp \left( -\frac{m\sigma_j^2}{\Omega j} \right) \sum_{k=0}^{m-1} \frac{1}{k!} \left( \frac{m\sigma_j^2}{\Omega j} \right)^k} \, d\gamma_j
$$

(11)

where $Q(\cdot)$ is the complementary error function

$$
Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} \exp \left( -\frac{u^2}{2} \right) du = \frac{1}{2} \text{erfc} \left( \frac{x}{\sqrt{2}} \right).
$$

**B. S + I Selection Diversity**

In an S + I selection scheme, the branch with the largest amplitude of the received composite signal is chosen for data recovery [see Fig. 1(b)]. Then, the average probability of bit error is given by

$$
P_b = 1 - \sum_{j=1}^{M} \text{Prob} [\zeta_j > 0 | b_0 = +1, \zeta_j > |\zeta_i|, i \neq j]
$$

and

$$
= 1 - \sum_{j=1}^{M} \int_{0}^{\infty} \int_{0}^{\infty} g_k(\zeta_j) \prod_{i=1, i \neq j}^{M} \left[ \int_{\zeta_i}^{\infty} f_k(\zeta_i) d\zeta_i \right] d\zeta_j.
$$

(13)

Assuming that $b_0 = +1$ was transmitted, the random variable $\zeta_j$ is the sum of a Gaussian and a Nakagami-distributed random variable. Therefore, the pdf of $g_k(\zeta)$ can be easily obtained through convolution of their individual density functions

$$
g_k(\zeta_j) = \int_{0}^{\infty} \frac{1}{\sqrt{2\pi}} \exp \left( -\frac{(z_j - x)^2}{2} \right) \frac{2m^m x^{2m-1}}{\Gamma(m) \Omega_j^{2m}} \cdot \exp \left( -\frac{m\sigma_j^2}{\Omega_j^2} \right) dx,
$$

and

$$
\zeta_j = \sqrt{\frac{\Omega_j}{\sigma_j^2}} \cdot D_{-2m} \left( -\frac{\sigma_j^2}{2m + \Omega_j} \right), \quad m \in \left\{ \frac{1}{2}, 1, \frac{3}{2}, 2, \cdots \right\}.
$$

(15)

After some mathematical manipulations, (15) can be rewritten as [see the Appendix]

$$
g_k(\zeta_j) = \frac{2m^m \Gamma(2m)}{\sqrt{2\pi} \Gamma(m) (2m + \Omega_j)^m} \exp \left( -\frac{\zeta_j^2}{4(2m + \Omega_j)} \right)
$$

$$
\cdot D_{-2m} \left( -\frac{\sigma_j^2}{2m + \Omega_j} \right), \quad m \in \left\{ \frac{1}{2}, 1, \frac{3}{2}, 2, \cdots \right\}
$$

(16)

where $D_{-2m}(z)$ is the function of the parabolic cylinder [16] of order $\nu$ and argument $z$. It is worth noting that $D_{-2m}(z)$
can be computed efficiently using the following recurrence relationship
\[ D_{p+1}(z) - zD_p(z) + pD_{p-1}(z) = 0 \] (17)
with \( D_{-1}(z) = \frac{1}{\sqrt{\pi}} \exp\left(\frac{z^2}{4}\right) \text{erfc}\left(\frac{z}{\sqrt{2}}\right) \), and \( D_{-2}(z) = \exp\left(-\frac{z^2}{4}\right) - z \exp\left(\frac{z^2}{4}\right) \sqrt{\frac{2}{\pi}} \text{erfc}\left(\frac{z}{\sqrt{2}}\right) \).

A numerical integration (Gaussian quadrature [17]) technique is applied to compute the average probability of bit error described in (14).

C. Maximum-Ratio Diversity

Maximum-ratio combining is known to be optimum in the sense that it yields the best statistical reduction of fading in any linear diversity combiner. A number of researchers have analyzed the performance of maximal-ratio diversity systems over Nakagami fading channels (e.g., [10], [18], [19]). It is worth noting, however, that the approximate BER expression found in [10] is only accurate if the discrepancy between the mean signal strengths are small (exact for equal mean signal strength case). By contrast, the formula derived in this paper is exact even if the mean signal strengths have a large variance, but is limited to integer \( M \).

In this technique, the \( M \) diversity branches are first cophased and then weighted in proportion to their signal level before summing. The output of the maximum-ratio combiner can be expressed as a single decision variable in the form
\[ U = \left( \sum_{j=1}^{M} \frac{\alpha_j}{\sigma_j^2} \right) b_0 + \sum_{j=1}^{M} \frac{\alpha_j \eta_j}{\sigma_j^2}, \] (18)

For a fixed set of \( \{\alpha_j\} \), the decision variable \( U \) is Gaussian with mean \( \mathbb{E}[U] = \sum_{j=1}^{M} \frac{\alpha_j^2}{\sigma_j^2} \). Subsequently, the average probability of bit error can be expressed as
\[ P_b = \int_0^\infty Q(\sqrt{\gamma}) f_{\gamma \text{nc}}(\gamma) d\gamma 
= \sum_{j=1}^{M} \sum_{k=1}^{\infty} \frac{(2k-1)!}{2^{k+1} k!} \frac{(m+1)F_k(k, k+1, -\frac{m}{\sigma_j^2})}{\Gamma(k+\frac{1}{2})} 
= \sum_{j=1}^{M} \delta \left( k, k+1, \frac{1}{2} \right) \left[ F_k(k, k+1, -\frac{m}{\sigma_j^2}) \right]^i, \] (19)

where \( \mu_j = \sqrt{\gamma_j/(2m+\gamma_j)} \), and \( A_{jk} \) is defined as
\[ A_{jk} = \binom{m}{k} \frac{d^n}{dx^n} \left[ \prod_{i=1}^{M} \frac{(1-x_i^2/m)^i}{(1-x_i^2/m)^{i-k}} \right] |_{x=\mu_j}, \] (20)

If we assume that \( \alpha_j \) and \( \eta_j \) \( j = 1, 2, \ldots, M \) are statistically mutually i.i.d. Nakagami and Gaussian random variables, respectively, then (19) can be simplified as
\[ P_b = \frac{1}{2} (1 - \mu) \left( \sum_{i=0}^{mM-1} \left( \sum_{j=1}^{M} \alpha_j \eta_j \right)^i \right)^i, \] (21)

where \( \mu = \sqrt{\gamma/(2m+\gamma)} \), and \( \gamma \) is the mean SNIR (which is the same on every branch). For the particular case of \( M = 1 \) and \( M = 1 \), (21) reduces to the familiar expression for \( P_b \) in a Rayleigh fading channel with no diversity. Also notice that when \( m = 1 \), the BER for \( M \)-diversity maximum-ratio combining in Rayleigh fading is equivalent to the expression given in Proakis [14, eq. (14-4-15)].

IV. NUMERICAL RESULTS

In this section, we provide some representative numerical curves illustrating the performance of a DS/CDMA network over a Nakagami multipath fading channel, based on the analytical results derived in the preceding sections.

First consider Fig. 2, which compares the BER performance of BPSK signals with different combining techniques and varying diversity order, \( M = \{1, 2, 3, 4\} \), over a Rayleigh fading channel. Obviously all the diversity combining methods yield the identical result for a single diversity branch. We have also verified that when the average signal power is zero [i.e., \( \gamma_b = 0 \) or \( \gamma_b \) (in decibels) \( \rightarrow -\infty \)], the probability of bit error is 0.5 (not shown in this figure), which is anticipated for BPSK signals. It is also apparent from this figure that the largest diversity gain is obtained using two-branch diversity and diminishing returns are obtained with increasing order of \( M \). This is typical for all diversity techniques. Another interesting point to note is that the \( S + 1 \) selection rule...
outperforms the traditional selection scheme. The explanation for this phenomena has been furnished in [4]. As an example, the differences in required SNR per bit predicted by the S + I selection model with respect to the conventional selection scheme for dual-diversity and fourfold diversity systems are 0.9 and 1.4 dB, respectively, at an average BER of $10^{-3}$. As we can see, the difference becomes more evident with a larger number of diversity branches. This may be attributed to the increased number of choices among statistically independent (Gaussian) noise samples [4].

Fig. 3 depicts the bit-error performance for BPSK signals over a Nakagami fading channel with fading figure $m = 2$, as a function of mean received SNR. Similar trends as in the Rayleigh fading case are observed here, except that the results are slightly better. Also notice that the results in these figures clearly illustrate the advantage of diversity as a means of overcoming the severe penalty in SNR/bit (power consumption) caused by fading. A few important conclusions can be drawn by comparing Figs. 2 and 3. First, the fade distribution affects the diversity gain. The relative advantage of diversity is greater for Rayleigh than Nakagami ($m > 1$) fading because as the fading figure increases, there is less difference between the instantaneous receiver SNR on the various diversity branches. However, the performance is always better with Nakagami ($m > 1$) fading than Rayleigh fading for a given average received SNR and diversity order. Next, the discrepancy between the SINR and S + I selection systems becomes more pronounced in environments that have strong specular paths (large $m$). Finally, it is observed that the performance of an S + I scheme is much closer to the maximum-ratio diversity than the conventional selection scheme for small diversity orders. This observation becomes more noticeable in good channel conditions. However, the reverse is true when the number of diversity branches grows (refer to Fig. 6).

It is worth noting that the results presented in Figs. 2 and 3 are directly applicable for noise-limited environments, whereas Figs. 4 and 5 correspond to the interference limited scenarios. For the Nakagami multipath fading model, we assume that the number of resolvable paths $L$ is three, having relative path strengths (0 dB, $-9.5$ dB, $-12$ dB) as suggested in [9]. This three-ray model (based on channel measurements conducted in downtown Ottawa and at the Communications Research Center) is applicable for system bandwidths of 5 MHz in an urban outdoor microcellular system or 20 MHz.
in an indoor microcellular system. Figs. 4 and 5 illustrate the
average BER performance of a microdiversity DS/SS multiple-
access network in different fading environments. We arbitrarily
choose the average received SNR of the first arriving path in
the absence of multiuser and self-interference, \((E[\hat{r}^2]E_b/N_0)\),
and processing gain values to be 15 and 63 dB, respectively.
Once again, the largest diversity gain is achieved with two-
branch diversity and diminishing returns are realized with
increasing \(M\). Other main conclusions drawn from Figs. 2 and
3 are also apparent by looking at these figures.

Fig. 5 shows that the dual diversity S + I system supports
almost as many active users as the optimum linear diversity
combiner with the same order at the average BER of 10^{-3}
or lower. Also notice that the same qualitative result can be
obtained from Fig. 3 in terms of average SNR/bit. This is par-
ticularly interesting in that the dual-diversity systems are by far
the more common in current applications. However, for higher
orders of the diversity, maximum-ratio combiner remains optimum
in terms of performance. Next in Fig. 6, we illustrate the
performance trend for three diversity combining techniques, as
the number of diversity branches increases. To create this plot,
an average SNR/bit of 7 dB was chosen as being representative
of the mobile communications environment. Notice that when
fading becomes lighter, the performance improvement with the
traditional selection diversity saturates much faster than
the S + I selection or maximum-ratio combining counterparts,
as the diversity order grows.

We shall now describe the effects of unequal mean sig-
nal strengths among the statistically independent diversity
branches on the error performance. To facilitate this analysis,
we introduce a simple tolerance model that adds a small
amount of perturbation \(\varepsilon\) to the average sum of the SINR \(\overline{\gamma}\).

Consequently, the mean SINR of the \(j\)th branch is given by

\[
\overline{\gamma} = \begin{cases}
\overline{\gamma}_j, & \text{if } j = \lfloor M/2 \rfloor \quad \text{and } M \text{ odd} \\
(1 - \varepsilon)\overline{\gamma}_j, & \text{if } j < \lfloor M/2 \rfloor \quad \text{and } M \text{ odd} \\
(1 + \varepsilon)\overline{\gamma}_j, & \text{if } j > \lfloor M/2 \rfloor \\
(1 - \varepsilon)\overline{\gamma}_j, & \text{if } j \leq M/2 \quad \text{and } M \text{ even}
\end{cases}
\]

where \(\overline{\gamma}_j = \Omega_j/\sigma_j^2\) and \(\overline{\gamma} = (\sum_{j=1}^{M} \overline{\gamma}_j)/M\).

It is important in our study to investigate the effect of variations
in mean signal strengths among the statistically independent
diversity branches are small. However, when the variation
is extremely large, a very noisy branch can easily upset the
system performance because it introduces ambiguity in the
selection process. Fortunately in practice, the deviation in mean signal strength is relatively small for microdiversity
reception (assumption of constant noise power in all diversity
branches is valid for most instances). Consequently, the
proposed S + I selection technique will always perform better
than conventional selection diversity when realistic practical
conditions are considered. But careful consideration should be
placed in case of macrodiversity reception because the above
assumption may become void.

Comparison between Tables I and II reveals an interesting
insight about the selection diversity systems. It was observed
that the S + I selection scheme is less susceptible to variations
in the mean received signal power levels compared to the
fluctuations of the average long-term noise power. In contrast,
the conventional selection diversity performs identically to the
variations in \(\sigma^2\) or \(\Omega\), which is anticipated. Figs. 7 and 8 depict
the error performances of the dual diversity receiver structures
(with different combining rules and unbalances in the mean
received signal strengths) for different fading environments. It
is apparent from these plots that the S + I selection system
can tolerate larger deviation of \(\sigma^2\) when the fading becomes
lighter. Although power control error (variations in \(\Omega\)) seems
to affect the performance of an S + I system more drastically
at higher \(m\), it is not a major concern since the perfor-
TABLE I
AVERAGE ERROR PROBABILITY TAKING INTO ACCOUNT THE VARIATION IN THE LONG-TERM AVERAGE NOISE POWER $\sigma_n^2$ AMONG THE STATISTICALLY INDEPENDENT DIVERSITY BRANCHES. IT IS ASSUMED THAT THE MEAN SNR/BIT $\tau_n = 7$ dB, AND $\sigma_n^2$ ARE IDENTICAL ON EVERY DIVERSITY BRANCH

<table>
<thead>
<tr>
<th>Diversity Order</th>
<th>Variation in $\bar{\gamma}_j$ due to $\sigma^2$</th>
<th>Average Bit-Error Probability, $P_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$m = 1$ (Rayleigh)</td>
</tr>
<tr>
<td></td>
<td>$\sigma$</td>
<td>SNIR</td>
</tr>
<tr>
<td>$M=2$</td>
<td>0 %</td>
<td>9.669 x 10^{-3}</td>
</tr>
<tr>
<td></td>
<td>±10 %</td>
<td>9.731 x 10^{-3}</td>
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<tr>
<td></td>
<td>±20 %</td>
<td>9.922 x 10^{-3}</td>
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<td></td>
<td>±30 %</td>
<td>1.026 x 10^{-2}</td>
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<td>±50 %</td>
<td>1.150 x 10^{-2}</td>
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<td>±75 %</td>
<td>1.499 x 10^{-2}</td>
</tr>
<tr>
<td>$M=3$</td>
<td>0 %</td>
<td>3.123 x 10^{-3}</td>
</tr>
<tr>
<td></td>
<td>±10 %</td>
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</tr>
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<td>3.193 x 10^{-3}</td>
</tr>
<tr>
<td></td>
<td>±30 %</td>
<td>3.283 x 10^{-3}</td>
</tr>
<tr>
<td></td>
<td>±50 %</td>
<td>3.607 x 10^{-3}</td>
</tr>
</tbody>
</table>

TABLE II
AVERAGE ERROR PROBABILITY TAKING INTO ACCOUNT THE VARIATIONS IN THE MEAN RECEIVED SIGNAL POWER $\Omega_j$. IT IS ASSUMED THAT THE MEAN SNR/BIT $\tau_n = 7$ dB, AND $\sigma_n^2$ ARE IDENTICAL ON EVERY DIVERSITY BRANCH

<table>
<thead>
<tr>
<th>Diversity Order</th>
<th>Variation in $\bar{\gamma}_j$ due to $\Omega = E[\sigma^2]$</th>
<th>Average Bit-Error Probability, $P_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma$</td>
<td>SNIR</td>
</tr>
<tr>
<td>$M=2$</td>
<td>±10 %</td>
<td>9.731 x 10^{-3}</td>
</tr>
<tr>
<td></td>
<td>±20 %</td>
<td>9.922 x 10^{-3}</td>
</tr>
<tr>
<td></td>
<td>±30 %</td>
<td>1.026 x 10^{-2}</td>
</tr>
<tr>
<td></td>
<td>±40 %</td>
<td>1.077 x 10^{-2}</td>
</tr>
<tr>
<td></td>
<td>±50 %</td>
<td>1.150 x 10^{-2}</td>
</tr>
<tr>
<td></td>
<td>±75 %</td>
<td>1.499 x 10^{-2}</td>
</tr>
<tr>
<td>$M=3$</td>
<td>±10 %</td>
<td>3.141 x 10^{-3}</td>
</tr>
<tr>
<td></td>
<td>±20 %</td>
<td>3.193 x 10^{-3}</td>
</tr>
<tr>
<td></td>
<td>±30 %</td>
<td>3.283 x 10^{-3}</td>
</tr>
<tr>
<td></td>
<td>±40 %</td>
<td>3.418 x 10^{-3}</td>
</tr>
<tr>
<td></td>
<td>±50 %</td>
<td>3.607 x 10^{-3}</td>
</tr>
</tbody>
</table>

V. CONCLUSIONS

The efficacy of selection diversity has been studied by comparing the performance of two receiver structures employing BPSK signaling over a Nakagami multipath fading channel. First, an accurate analytical expression to evaluate the performance of a practical S + I selection diversity scheme on various propagation channels has been presented.

...
Fig. 7. Effects of the variations in $\sigma^2$ and $\Omega$ on the BER performance of the SINR and $S+I$ selection systems, in a Rayleigh faded environment. $\gamma_1$ is fixed at 15 dB, and $M = 2$.

Fig. 9. Effects of unequal mean received signal strengths on the BER performance of a dual diversity SINR and $S+I$ selection systems. $\gamma_1$ is fixed at 5 dB, and fading figure $m = 2$.

An attractive feature of the $S+I$ selection rule is that it is much easier and cheaper to implement in practice. It has been shown that the new scheme outperforms the traditional selection diversity model in an equal mean signal strengths scenario. The difference becomes more evident with a larger number of diversity branches and in environments that do not experience severe fading. Next, we have examined the effects of unequal mean signal strengths on the BER performance. The discrepancy between the $S+I$ and SINR selection techniques diminishes as the variation in the mean signal strengths among the diversity branches becomes larger.

In this appendix, we describe the important intermediate steps involved in the transformation of (15) into (16). Let us denote $\bar{\gamma}_j = \Omega_j/\sigma_j^2$, then (15) can be restated as

$$g_c(z_j) = \frac{1}{\sqrt{2\pi}} \int_0^\infty \exp\left(-\frac{(z_j-x)^2}{2}\right) \frac{x^{2m} \exp(-z_j x/2)}{\Gamma(m) \sigma_j^2} \, dx$$

$$= \frac{2m^m}{\sqrt{2\pi} \Gamma(m) \sigma_j^2} \exp\left(-\frac{z_j^2}{2}\right) \int_0^\infty x^{2m-1} \exp\left(-\frac{2m + \bar{\gamma}_j}{2} x^2 - z_j x\right) \, dx.$$  

We recognize that the definite integral in (A.1) can be expressed in terms of the parabolic cylinder function using the following identity [16, p. 337]

$$\int_0^\infty x^\nu \exp(-ax^2 - cx) \, dx = (2b)^{-\nu/2} \Gamma(\nu) \exp\left(-\frac{c^2}{8b}\right) D_{-\nu}\left(\frac{c}{\sqrt{2b}}\right), \quad b > 0, \nu > 0$$  

(A.2)
and $D_{m-1}(\cdot)$ for any nonnegative integer $n$ is given by

$$D_{m-1}(z) = \sqrt{\frac{\pi}{2}} \frac{(-1)^n}{n!} \exp\left(-\frac{z^2}{4}\right) \frac{d^n}{dz^n} \left[ \exp\left(\frac{z^2}{2}\right) \operatorname{erfc}\left(\frac{z}{\sqrt{2}}\right) \right], \quad n = 0, 1, 2, \ldots, \tag{A.3}$$

After some algebraic manipulations, we obtain $g_k(z_j)$ as illustrated in (16). Further, we would like to point out that by computing the derivatives of the parabolic cylinder function, the recurrence relationship described in (17) can be easily verified. Consequently, $D_{m-1}(\cdot)$ can be computed efficiently using this recurrence relationship, with $D_1(z)$ and $D_2(z)$ obtained using (A.3).

For the particular case of $m = 1$ (Rayleigh fading channel), (16) reduces to

$$g_k(z_j) = \frac{2}{\sqrt{2\pi}(2+\gamma_j)} \exp\left(-\frac{z_j^2}{4(2+\gamma_j)}\right) \cdot D_2\left(-z_j \sqrt{\frac{\gamma_j}{2(2+\gamma_j)}}\right)$$

$$= \frac{2}{\sqrt{2\pi(2+\gamma_j)}} \exp\left(-\frac{z_j^2}{2}\right) \operatorname{erfc}\left[-z_j \sqrt{\frac{\gamma_j}{2(2+\gamma_j)}}\right]$$

which is equivalent to the expression given in [5].

ACKNOWLEDGMENT

The author would like to thank Prof. N. C. Beaulieu of Queen’s University, Kingston, ON, for correspondence, and he is also very grateful to Prof. V. K. Bhargava for his encouragement and guidance. The author would also like to thank the anonymous reviewers for their helpful comments and constructive suggestions.

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A. Annamalai (M’95), for a photograph and biography, see p. 1344 of the September 1999 issue of this TRANSACTIONS.